Fast Design of Repeat Ground Track Orbits in High-Fidelity Geopotentials*

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Abstract

The existence of families of periodic, repeat ground track orbits in full Geopotentials is demonstrated. The basic families are made of almost circular orbits except in the vicinity of the critical inclination (63.4/116.6 deg), where the eccentricity of the repeat orbits grows high for almost fixed inclination. Computation of specific repeat ground track orbits for mission designing purposes can be automated providing the nominal solution in a fast, straightforward way; this is illustrated with the computation of the TOPEX nominal orbit in a 140×140 truncation of the GRACE Gravity Model.

Introduction

Many missions for artificial satellites are devoted to Earth and climate observation such as ICESat, CryoSat, and TOPEX/Poseidon. These types of missions as well as a whole host of other applications including InSAR mapping missions and communication constellations benefit from the repeat ground track (RGT) geometry.

The procedure of mission design starts typically from the experiment requirements, which constrain the orbital parameters to a subset of limited values. Then, a first order repeat ground track design is achieved including J_2 to approximate the nominal solution. Further refinements of the orbital elements —usually in the presence of a medium degree zonal model, but sometimes including drag and lunar and solar perturbations— will provide the nominal orbit. The refinement procedure aerospace engineers normally use is based on interactive trial and error corrections that converge to a good nominal set of orbital elements; where "good" indicates that the satellite does not drift substantially from the repeat ground trace. This refinement is generally performed via small adjustments to the semimajor axes and the eccentricity in a manual iterative sequence.

However, it is known that periodic solutions of the artificial satellite problem exist for either zonal or full (zonal, sectoral and tesseral harmonics) gravity models [1, 2, 3, 4] when the problem is formulated in a rotating frame attached to the central body. These periodic orbits repeat exactly their ground track on the surface and, hence, are ideal candidates as

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nominal orbits for RGT missions. In addition, the technique provides the stability character and average orbital elements of the RGT orbit. Most importantly perhaps, the technique is simple to automate and mission designers can select from full families of RGT orbits.

Here, we extend this new technique for Earth applications. Our results include the higher order effects of a full gravity field and require no tuning. As a proof of concept we use the GRACE Gravity Model (GGM02C) and apply this technique to the computation of the TOPEX nominal orbit with a 10/127 RGT cycle. Thus, we compute the family of frozen, 10/127 repeat orbits in a moderate gravity field. We find high inclination, low eccentricity, stable, periodic orbits for all inclinations embraced by the critical inclinations, where high eccentricity periodic orbits exist with frozen perigee. Then we propagate the initial conditions of selected repeat orbits in a higher order (140×140) gravity field. After differential corrections we find that, indeed, the orbit remains periodic.

For mission designing tasks the whole procedure can be fastened as follows. First, a simple analytical approximation provides initial conditions of the approximated nominal RGT orbit. These initial conditions are amenable to improvement with differential corrections until finding a periodic orbit of a low order, say 9×9, gravity model. The fast computation of the corresponding family of periodic orbits of this model reveals the details of the real phase space and permits to chose better initial conditions. Then, new differential corrections converge to a true repeat orbit of a higher order gravity model. Finally, if required, the solution can be re-adjusted by propagating the family of periodic orbits of the higher order gravity problem.

Periodic orbit computation in full Geopotentials

In an Earth centered frame with the x axis pointing to Greenwich meridian, the z axis to the pole and the y axis completing a direct frame, the equations of motion are

$$\ddot{x} - 2\omega\,\dot{y} = \frac{\partial\Omega}{\partial x}, \qquad \ddot{y} + 2\omega\,\dot{x} = \frac{\partial\Omega}{\partial y}, \qquad \ddot{z} = \frac{\partial\Omega}{\partial z}, \tag{1}$$

where $\omega = 360.9856235\,\mathrm{deg/day}$ is the Earth's rotation rate [5] and Ω is the potential function

$$\Omega = (\omega^2/2)\,(x^2 + y^2) - V \tag{2}$$

with

$$V = -\frac{\mu}{r}\left[1 + \sum_{j\geq 2}\left(\frac{\alpha}{r}\right)^j \sum_{k=0}^{j} (C_{j,k}\cos k\lambda + S_{j,k}\sin k\lambda)\,P_{j,k}(\sin\varphi)\right] \tag{3}$$

where μ is the Earth's gravitational parameter, the scaling factor α is the equatorial radius of the Earth, $P_{j,k}$ are the Legendre functions in the sine of the latitude φ, λ is longitude, and $C_{j,k}$ and $S_{j,k}$ are the gravitational coefficients of the potential. In the rotating frame V is time independent and the equations of motion accept the integral

$$2\Omega - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = C \tag{4}$$

System (1) is not integrable —even in the simplest case in which all coefficients vanish except $C_{2,0}$ [6]. However, one can obtain valuable information by computing particular solutions to it. Specifically, periodic orbits are solutions that are known for all the time and whose stability properties are easily determined. Further, the existence of an integral, Eq. (4), guarantees that periodic orbits are grouped in families that determine the skeleton of the dynamical system [7].

As the main perturbation from the Keplerian motion around the Earth is due to the oblatenes coefficient $C_{2,0}$ that is of the order of 10^{-3}, in most cases it is enough to compute differential corrections to a rough Keplerian approximation to obtain a true periodic orbit in the full Geopotential Eq. (3).

GRACE Gravity Model

The GGM02C Model is a high resolution global gravity field model estimated with data from the GRACE satellites combined with other terrestrial gravity information [8]. The complete model includes 200×200 spherical harmonic coefficients and the $C_{2,0}$ term is constrained to represent the accurate long-term mean value. The GGM02C model is publicly available[1] and is currently accepted as the most accurate model to date for high precision orbit determination applications.

Basics on differential corrections computation

Let $\boldsymbol{x}$ be of dimension m and let

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, t) \tag{5}$$

be an m-dimensional differential system to which a solution

$$\boldsymbol{x} = \boldsymbol{x}(t) \tag{6}$$

is known for certain initial conditions. Assume also that $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{\xi}$ is another solution to Eq. (5). Therefore,

$$\dot{\boldsymbol{x}} + \dot{\boldsymbol{\xi}} = \boldsymbol{F}(\boldsymbol{x} + \boldsymbol{\xi}, t).$$

If we assume $\boldsymbol{y}$ to be close enough to $\boldsymbol{x}$ for all t, the displacements $\boldsymbol{\xi}$ will be small. Then, a Taylor series development of the force function gives rise to the variational equations

$$\dot{\boldsymbol{\xi}} = J\,\boldsymbol{\xi} \tag{7}$$

where J is the Jacobian matrix of elements $\partial F_i/\partial x_j$, $(i, j = 1, \ldots, m)$.

The variational equations, Eq. (7), are a linear homogeneous system of ordinary differential equations whose general solution is given by a linear combination of m independent solutions. Thus,

$$\boldsymbol{\xi} = \sum_{i=1}^{m} C_i\,\boldsymbol{\xi}_i(t), \tag{8}$$

where C_i are m arbitrary constants and $\boldsymbol{\xi}_i = \boldsymbol{\xi}_i(t)$ form a fundamental system to Eq. (7).

Now, assume that after a time $t = T_0$ the known solution $\boldsymbol{x} = \boldsymbol{x}(t)$ is periodic or almost periodic, and that we want to determine the values of C_i that make $\boldsymbol{y}$ also to be periodic with, probably, a new period $T = T_0 + \Delta T$. Then, for the theorem on existence and

[1] http://www.csr.utexas.edu/grace/gravity/ [cited: Jan 1, 2008]

uniqueness of solutions to initial value problems, it is enough for the periodicity condition $\boldsymbol{y}(t) = \boldsymbol{y}(t+T)$ to be fulfilled that

$$\boldsymbol{y}(0) = \boldsymbol{y}(T_0 + \Delta T). \tag{9}$$

For close solutions to $\boldsymbol{x}$ the correction ΔT is small and, neglecting higher order quantities and noting that $\partial\boldsymbol{x}/\partial T_0 = \boldsymbol{F}(\boldsymbol{x}(T_0), T_0)$, the periodicity condition Eq. (9) is written

$$\boldsymbol{x}(0) + \boldsymbol{\xi}(0) = \boldsymbol{x}(T_0) + \boldsymbol{F}(\boldsymbol{x}(T_0), T_0)\,\Delta T + \boldsymbol{\xi}(T_0)$$

or

$$\boldsymbol{F}(\boldsymbol{x}(T_0), T_0)\,\Delta T + \sum_{i=1}^{m} \left[\boldsymbol{\xi}_i(T_0) - \boldsymbol{\xi}_i(0)\right] C_i = \boldsymbol{x}(0) - \boldsymbol{x}(T_0) \tag{10}$$

Therefore, the problem of computing a periodic solution close to a known periodic or almost periodic solution is reduced to, first, computing a fundamental system to Eq. (7) and, then, solving the linear system Eq. (10) for the unknowns C_i and ΔT. Remarkably, when the fundamental system to Eq. (7) is computed using the identity matrix $\boldsymbol{I}_m$ as initial conditions, the constants C_i result to be the required corrections to the initial conditions $\boldsymbol{x}(0)$ that determine the initial conditions $\boldsymbol{y}(0)$ of the new periodic solution.

There are several matters that may make solving Eq. (10) difficult, especially when the equations of motion Eq. (5) accept integrals. The linear system has fewer equations than unknowns; the matrix to invert may be singular or bad conditioned; etc. These are problems of linear algebra that may be approached in a variety of forms, and there is a wealth of algorithms for the continuation of families of periodic orbits based on the computation of differential corrections. The interested reader is referred to the literature and we only mention that the computations in this paper were double checked using the algorithms described in [4] and [9].

Families of periodic orbits

Periodic orbits of Hamiltonian system are not isolated but appear grouped in families. Further, for the Earth, it has been shown that RGT orbits are members of families of three-dimensional periodic orbits in the rotating frame that appear as vertical bifurcations of the family of (planar) equatorial orbits [2] —the straightforward computation of the bifurcated families just requiring a small displacement of the initial conditions of the critical orbit in the vertical direction [10]. But, the simple non vanishing of the harmonic coefficient $C_{3,0}$ breaks the equatorial symmetry of the Geopotential, thus preventing the existence of planar solutions.

However, due to the magnitudes of the harmonic coefficients of the Geopotential, there exist periodic orbits very close to the equatorial plane, and families of RGT orbits indeed bifurcate from critical stability orbits of the family of (almost planar) almost equatorial orbits. The procedure of computing the bifurcated families is not so simple as in the equatorial symmetry case and will be discussed eventually. On the contrary, for mission designing purposes it is not necessary to compute whole families but, perhaps, a small part of them. Then, the procedure described below can be used as a shortcut for exploring the real phase space of nominal RGT orbits.

Practical approach

Geographical considerations related to a satellite mission may bound the inclination and eccentricity of the nominal orbit. Then, the required repetition cycle determines its semimajor axis and period; on the contrary, physical or technical reasons may constrain the altitude of the orbiter to some range (and, therefore, its semimajor axis and period), from which an adequate repetition cycle must be selected.

Once the initial requirements are fixed, a rough approximation of the nominal orbit can be obtained from simple analytical approximations [11] (see also [12], p. 45 and ff.) or in some cases based only on two body dynamics. Then, differential corrections produce a "true" (up to certain numerical precision) periodic orbit. However, the procedure of introducing differential corrections trying to find periodicity may shift slightly the elements from their reference values. Then, after finding the initial RGT periodic orbit, moving over the family for variations of the Jacobi constant may be required.

We illustrate the procedure using the orbital parameters of The Ocean Topography Experiment (TOPEX) mission, a joint project of NASA and the French space agency CNES, taken from [13]. Namely, a nearly circular reference RGT orbit covering 127 orbits over 10 sidereal days with an inclination of ~ 66 deg.

TOPEX 10/127 RGT family

From initial elements $d = 10$ days, $n = 127$ cycles, $e = 0$, $I = 90$ deg, $\Omega = 0$, a rough analytical approximation including $C_{2,0}$ and $C_{3,0}$ produce an orbit with $a = 7740.258\,\mathrm{km}$, $e = 0.00096$, $g = 90\,\mathrm{deg}$, $I = 66\,\mathrm{deg}$ that repeats its ground track after, $T = 861641.3092\,\mathrm{s}$. However, in the Earth's fixed frame, corresponding initial conditions $(\boldsymbol{r}, \dot{\boldsymbol{r}})$ of this orbit do not achieve periodicity in a real Geopotential, resulting in a miss distance $\delta \sim 0.003$, where

$$\delta = \mathrm{Max}\left\{ \frac{\|\boldsymbol{r}(T) - \boldsymbol{r}(0)\|}{\|\boldsymbol{r}(0)\|}, \frac{\|\dot{\boldsymbol{r}}(T) - \dot{\boldsymbol{r}}(0)\|}{\|\dot{\boldsymbol{r}}(0)\|} \right\}.$$

After applying differential corrections in a 20×20 truncation of GGM02C, we improve the initial conditions until reaching a "true" (miss distance $\delta < 10^{-11}$) RGT periodic orbit with period $T = 861640.6277\,\mathrm{s}$ and average orbital elements $a = 7740.261\,\mathrm{km}$, $e = 0.00114$, $I = 89.9984\,\mathrm{deg}$, $g = 89.9985\,\mathrm{deg}$, $\Omega = 0.0054\,\mathrm{deg}$.

Once a periodic orbit has been computed, we easily continue the whole family of periodic orbits that repeat the ground track after 127 cycles either for increasing or decreasing values of the Jacobi constant —which results in decreasing or increasing inclinations, respectively, when starting near the polar case. Besides, we obtain the stability character of each periodic orbit as a side effect of the differential correction procedure, because it is derived from the eigenvalues of the state transition matrix at the end of one period. For Hamiltonian systems with three degrees of freedom, as Eq. (1), two stability indices b_1, b_2, are normally used [14]. The condition $b_{1,2}$ real and $|b_{1,2}| < 2$ applies for stability in the linear approximation.

Figure 1 shows the stability curves of this family of 10-127 RGT periodic orbits. The family is generally made of stable orbits and only exists in the region $C \in (0.762058, 0.884604)$, where C is expressed in internal units such that the Earth gravitational constant and its equatorial radius are equal to one. At each end the family reflects over itself and both reflected parts (not appreciated at the scale of the plot) are made of unstable orbits.

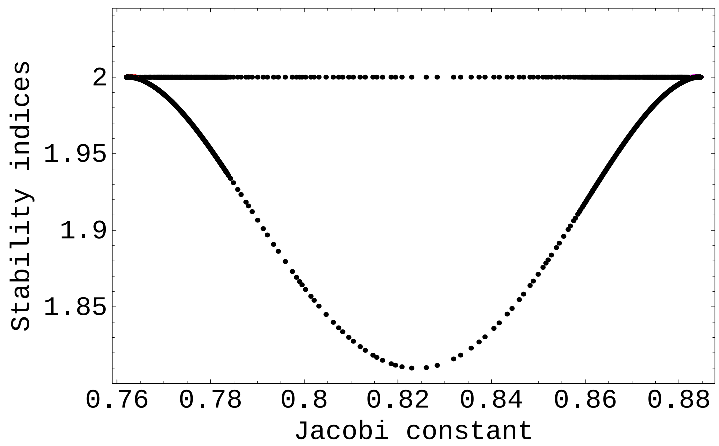

Figure 1: Family of 10-127 RGT periodic orbits: stability curves.

Remarkably enough, we always find one stability index with the degenerate value $b_1 = 2$. This behavior is analogous to the zonal problem (the problem in which only zonal harmonics are considered in the Geopotential), which accepts the polar component H of angular momentum as integral of the motion and, therefore, is separable: The planar motion in the rotating meridian of the satellite separates from the motion of this plane. Periodicity in three dimensions requires periodicity of each motion separately and, further, commensurability between both frequencies, and it occurs in a variety of cases [1]. Therefore, we conjecture that this kind of degeneracy, commonly associated to long repetition cycles [4, 15], might be a consequence of a sub-periodicity in the motion.

Since the separability of the zonal problem makes evident in cylindrical coordinates (ρ, z, λ) [16], we investigate the evolution of the orbit in these coordinates, in which $H = \rho^2\dot{\lambda}$ is the conjugate momentum to λ. Thus, a cursory inspection of the motion shows that z repeats its path after a nodal period, and ρ and H after half of it. Then, the motion in the rotating meridian of the orbiter —motion in the (ρ, z) plane, in cylindrical coordinates— seems to enjoy a 1/127th sub-periodicity of the full period, illustrated in Fig. 2.

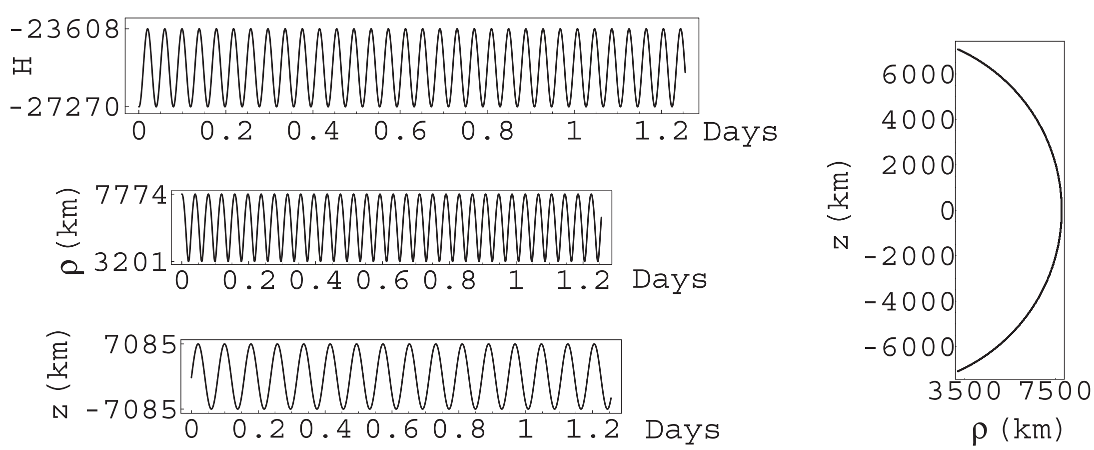

Figure 2: Sub-periodic behavior in a rotating meridian ($e = 0.00058$, $I = 114.25$ deg).

However, as shown in Fig. 3, a thorough check reveals that the sub-periodicity is only apparent: while the path of the orbiter in the rotating meridian is clearly mimicked each nodal period, the tesserals influence slightly affects periodicity.[2] Further research will disclose whether the degeneracy of the stability index is real or just a consequence of the numerical precision reached in the computation of the state transition matrix after such long periods.

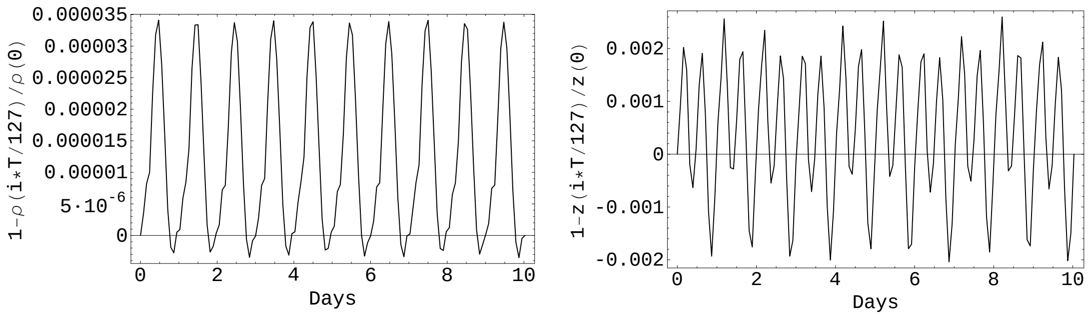

Figure 3: Relative sub-periodicity error ($i = 1, \ldots, 127$).

Another effect of the presumed singularity is that it affects the matrix inversion required by most differential corrections algorithms. A singular value decomposition procedure showed good results in constructing the generalized inverse of the matrix [17, 4].

Figure 4 shows the evolution of the family in an inclination-eccentricity diagram [18]. Reflections in the Jacobi constant roughly coincide with reflections in inclination, and the family is confined to the range of high inclination orbits encompassed by the direct and retrograde critical inclinations, $I = 63.435$ and $I = 116.565$ deg respectively. The lower eccentricity orbits between both critical inclinations are stable, and the higher eccentricity orbits with direct or retrograde (almost) critical inclination are unstable. Since elliptic orbits with $e > 0.17$ impact the surface of the Earth we do not continued the family for highly elliptic orbits.

Note in Fig. 4 the dips in eccentricity between 64.6 and 72.3 deg and between 107.5 and 115.3 deg, in which the eccentricity falls below 0.001. These dips could be good options for RGT missions because of their favorable inclinations and stability: it is not a surprise that TOPEX orbit, with an inclination of 66 deg, is placed in the direct inclination dip so close to the minimum $e = 0.0006$ that occurs at $I = 65.8$ deg.

Further, the RGT periodic orbits are frozen orbits with argument of the perigee that averages to 90 deg between the very low eccentricity orbits at 65.8 and 114.2 deg. Out of this interval the argument of the perigee averages to 270 deg either for the elliptic or almost circular orbits.

Remark that the instantaneous perigee circulates for most, if not for all, very low eccentricity orbits. However, its argument clearly averages to the same value each nodal period. That is exactly what we mean when we talk about the frozen perigee of very low eccentricity orbits, the value of which we compute from $\langle g \rangle = \arctan(\langle e \sin g \rangle / \langle e \cos g \rangle)$. Of course, in the transition from $\langle g \rangle = 90$ to $\langle g \rangle = 270$ deg that occurs near each of the eccentricity dips

[2]In fact, the influence of resonant tesseral harmonic coefficients is very important in lower resonances, a case in which we did not find this supposed degeneracy to happen.

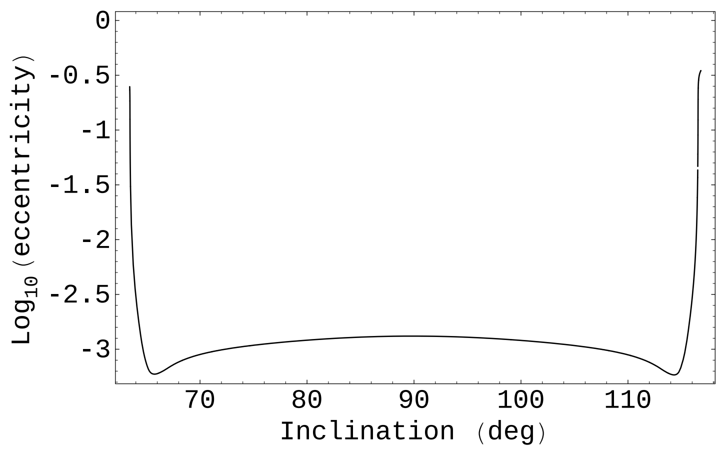

Figure 4: Family of 10-127 RGT periodic orbits: inclination stability diagram.

there must be at least one orbit, not necessarily the lowest eccentricity orbit, with undefined (averaged) argument of perigee, averaging half of the time to 90 deg and the other half to 270. Examples of these types of orbits are illustrated in Fig. 5, where we show the instantaneous evolution in the $(e\cos g, e\sin g)$ plane of very low eccentricity orbits ($e \sim 0.0006$), of the family of 10-127 RGT, periodic orbits of a 20×20 Geopotential. From left to right, the (averaged) inclinations are $I = 113.51$, $I = 114.24$, and $I = 114.77$, respectively.

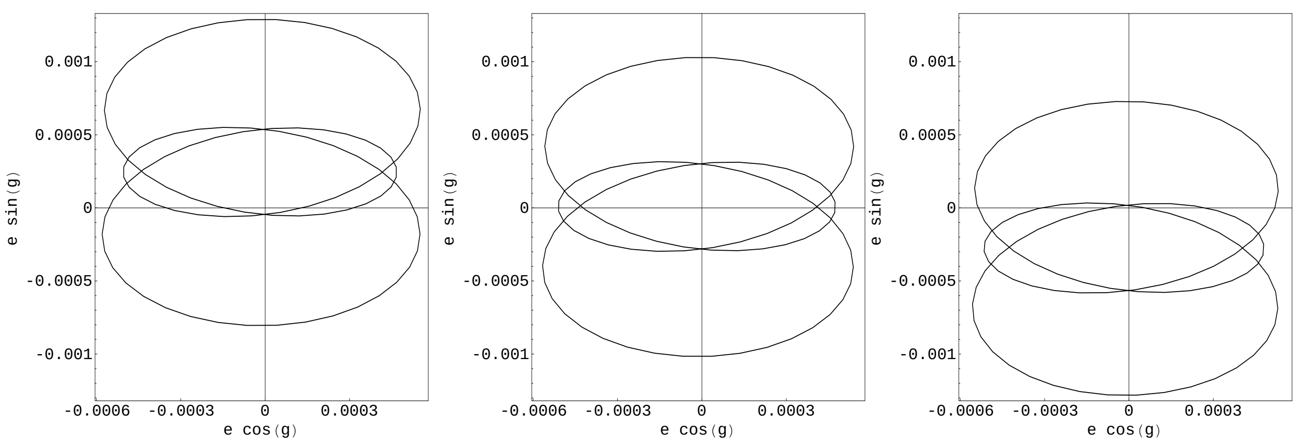

Figure 5: Orbits with instantaneous circulating perigee whose argument averages to $g = 90$ deg (left), undefined (center), and $g = 270$ deg (right).

TOPEX orbit: direct computation

The continuation of the natural family of RGT periodic orbits for variations of the Jacobi constant in the 20×20 gravity field above reveals the main features of the phase space around the Earth. Thus, we can selected the reference orbit from a computed family in a medium gravity field and, then, differentially correct to compute the RGT periodic orbit in a high fidelity Geopotential. However, the procedure of computing differential corrections may affect the inclination making it necessary to slightly continue the family in the desired direction for obtaining the desired orbit in the higher order field.

Alternatively, one can choose the direct computation of a specific *d*-*n* RGT orbit in a high fidelity Geopotential without need of computing previously the whole or part of the family of *d*-*n* RGT orbits in a medium or low gravity field. Thus, for a 50×50 truncation of the GGM02C, by requiring a 10-127 RGT cycle, almost circular orbit at 66 deg of inclination we get an analytical approximation with miss distance $\delta_0 = 0.4\times 10^{-2}$ that after successive corrections converges into the sequence $\delta_0 = 0.1\times 10^{-3}$, $\delta_2 = 0.1\times 10^{-5}$, $\delta_3 = 0.5\times 10^{-11}$, to a periodic RGT orbit of the 50×50 field, its final orbital elements being very close to the TOPEX reference mean elements for the operational orbit [13], with initial conditions

```
-0.4396226316250480E+00  0.8567054972658054E+06
-0.7479425656222416E+04  0.1903622797297078E+04  0.
-0.5823215938530145E+00 -0.2285419684650048E+01 -0.6567933919757708E+01
```

where the first row gives the Jacobi constant $(-C/2)$ in internal units and period in seconds, and the following lines, in the body fixed frame, give the position (km) and velocity (km/s).

After new differential corrections we succeed to converge in a similar sequence (starting from $\delta_0 < 10^{-7}$) to a periodic orbit in a 140×140 gravity field:

```
-0.4396226315392065E+00  0.8567054972866251E+06
 0.6785302750836524E+04 -0.3677091332059861E+04  0.
 0.1123234753502430E+01  0.2073920775568003E+01  0.6568136302551559E+01
```

with average orbital elements $a = 7714.405$ km, $e = 0.00060$, $I = 66.009$ deg, $g = 90.12$ deg, $\Omega = 321.24$ deg; period $T = 856705.4972866$ s miss distance $\delta = 10^{-11}$; and stability indices $b_1 = 2$, $b_2 = 1.994053$.

The TOPEX nominal orbit is presented in Fig. 6. The evolution of its instantaneous orbital elements is shown in Figs. 7 and 8.

It is worth mentioning that our software implements Pine's nonsingular equations [19] for computing the Geopotential and its derivates, but with alternative recursions that avoid the numerical instabilities found in the evaluation of derived Legendre functions for higher order degree and order potentials [20]. We run our simulations in a PowerPC G4 1.25 GHz processor, in which a ten-days integration of a typical 10-127 RGT low eccentricity orbit plus a fundamental set of variations requires less than two seconds for a 9×9 truncation of the Geopotential, about six for a 20×20 truncation, ~ 37 seconds for 50×50, and ten minutes for the 140×140 gravity field. Usually, a step to converge to a periodic orbit of the family requires of five integration cycles (one in the predictor stage and four in the corrector), so the preceding numbers are roughly multiplied by five.

Finally, despite that our procedure only accounts for the Earths gravitational effects, we check the robustness of its solutions to typical dominant perturbations. In the limited number of test cases, we generally find that the nominal orbit suffers only a very small drift of the reference RGT.

Thus, the left plot of Fig. 9 shows the evolution after ten cycles (about three months) of the computed nominal RGT orbit, periodic in the 50×50 field, in four different ephemeris cases. When accounting for the torque-free motion of the Earth, or more precisely the polar motion, the ground track drift is of just a few hundred meters in the equator. Similar drifts are obtained when taking into account lunar or solar perturbations. The combined effect of

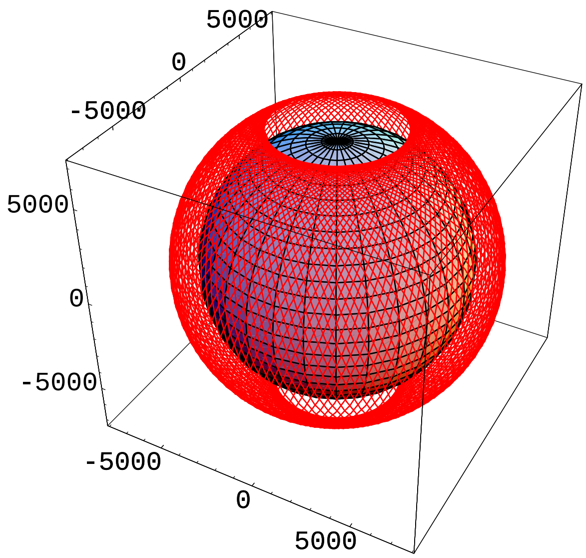

Figure 6: TOPEX nominal orbit (140×140 gravity field).

the polar motion and luni-solar perturbations introduces a drift in the equator that is less than one km in the above mentioned period. To briefly investigate the benefit of including the cross (sectoral and tesseral) terms, the right plot of Fig. 9 shows similar ephemeris propagations, only this time the reference RGT orbit is taken from a 50×0 zonal only model. Clearly, the removal of the cross terms in the reference calculations introduces a secular drift of nearly an order of magnitude greater than that caused by the polar motion and luni-solar perturbations.

In practice, the precise knowledge of the semimajor axis (mainly affected by atmospheric drag and radiation forces of body fixed origin and other model imperfections) is essential for effective ground-track control [13]. Therefore, the nominal solutions computed with our procedure may be desired to be further optimized in real models.

Conclusions

The computation of orbits for missions requiring repetition of the ground trace is amenable to be fully automated. The inclusion of the higher order effects of large Geopotentials is easily embraced in the procedure (sacrificing only computational time), and valuable stability information is inherited as a side effect. Furthermore, the procedure is not restricted to computing specific orbits and one can easily explore the close behavior of the nominal solution for variations of the orbital elements. Specifically, for a given repeat cycle, the simple inspection of the eccentricity-inclination diagram of the corresponding family of periodic orbits provides a crucial design trade necessary for the general mission design.

The method we advocate is not limited to Earth applications and works well for other two body problems like Mars, Venus, Mercury, or even irregularly shaped asteroids. Besides, the method is almost transparent to the inclusion of strong third-body perturbations as long as the orbiting body is synchronously locked with its primary.

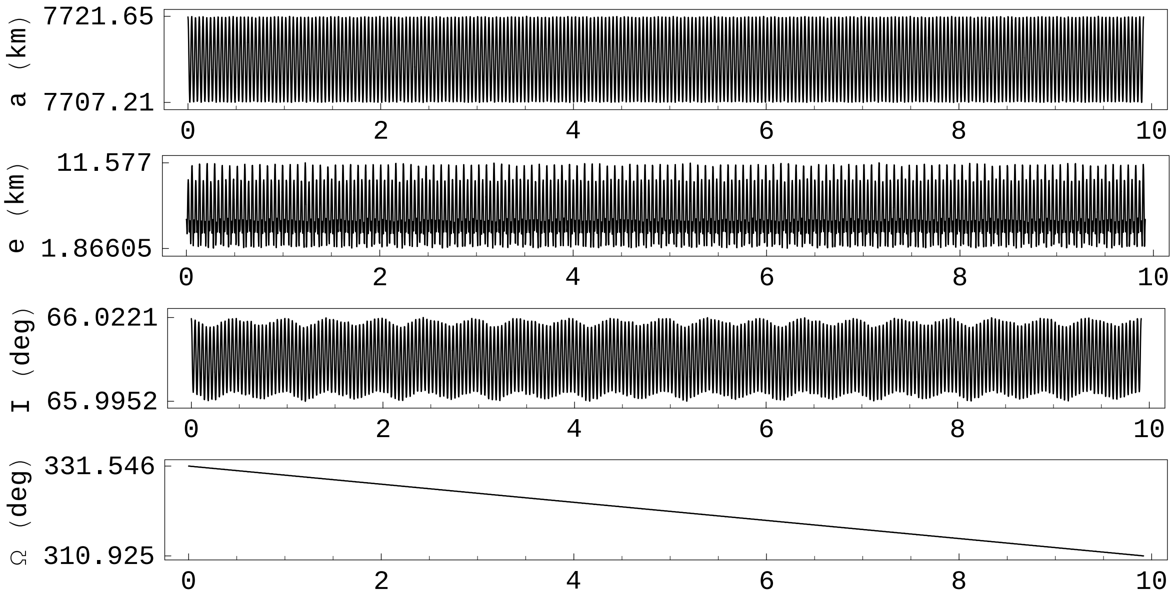

Figure 7: Evolution of TOPEX nominal orbit osculating elements along the $\sim$ 10 days period. From top to bottom: semimajor axes (km), eccentricity ($\times 10^{-4}$), inclination (deg), and argument of the node (deg).

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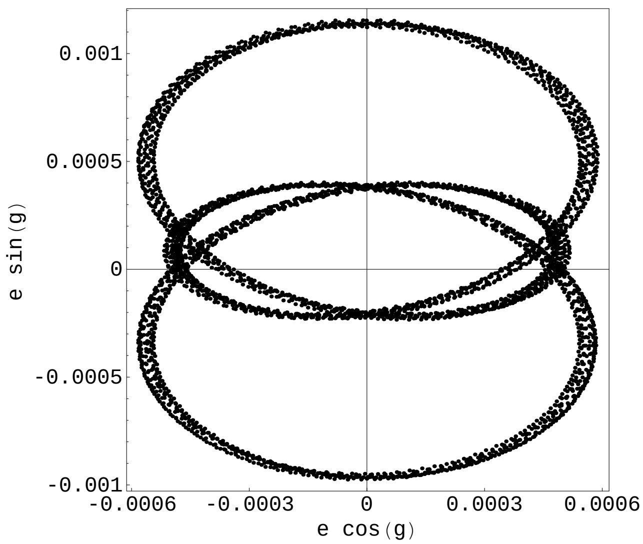

Figure 8: TOPEX nominal orbit osculating semi-equinoctial elements.

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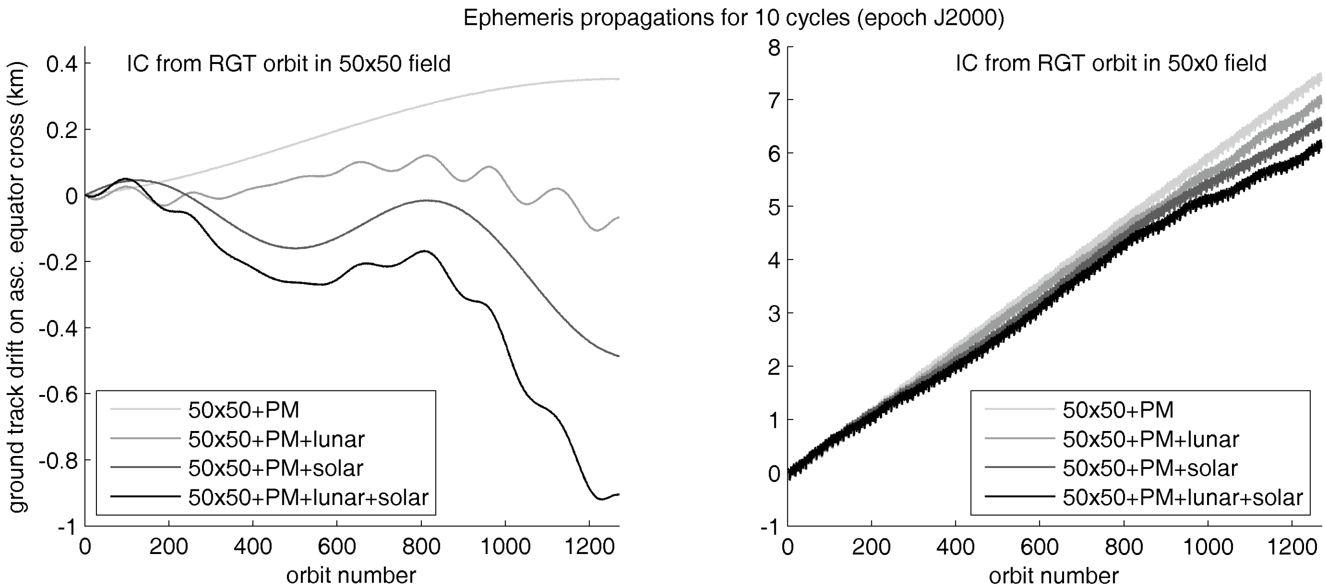

Figure 9: Evolution of representative TOPEX nominal RGT orbit ($I_0 = 66.17$ deg) in the ephemeris model.

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