

METHODS FOR DUAL-OBJECTIVE HIGH-ENERGY TOUR DESIGN

Yuri Shimane,* and Keidai Iiyama†

High-energy, multi-fly-by tours are a complex design task that typically arises in planetary moon tours, such as those of the Jupiterian or Saturnian systems. Inspired by this challenge, the 2022 edition of the Space Optimisation Competition (SpOC) organized by the Advanced Concepts Team included a problem involving a ΔV and time of flight optimization of a tour visiting the seven planets of the Trappist-1 star system. This work introduces the preliminary analyses and heuristics developed to tackle the multiobjective optimization problem, along with the results found for the Trappist-1 tour problem.

INTRODUCTION

Multi-fly-by trajectories with high incoming energy have been gaining significant attention over the past decades due to their applicability in moon-tour missions of multi-moon systems such as the Jupiter and Saturn systems. These high-energy tours can be challenging to design due to the combinatorial nature of the fly-by paths, the phasing of the moons, combined with the limited fuel, or ΔV budget, of the spacecraft.

To tackle this challenge, some analytical insights may be obtained through techniques such as Tisserand graphs¹ or V-infinity and Tisserand leveraging transfers (VILT/TILT).²⁻⁸ The end goal typically results in a global optimization problem, where both ΔV and time of flight, potentially among other objectives, are to be minimized. During the GECCO 2022 Space Optimisation Competition (SpOC),⁹ hosted by the Advanced Concepts Team (ACT) of the European Space Agency, one of the problems posed was designing such high-energy tours for visiting each of the seven planets of the Trappist-1 star system exactly once, with the objective of minimizing both ΔV and time of flight (TOF). While our visit to our stellar neighbor might not happen for another while, the challenges and lessons learned in solving this problem are directly applicable to the moon-tour problem, especially in cases where a rapid tour of all bodies of interest is desirable.

The problem organizers have formulated the tour design problem as a variant of the MGA-1DSM model. The MGA-1DSM model is a commonly used formulation for designing multi-gravity assist (MGA) transfers, including a single deep-space maneuver (DSM) during each leg.¹⁰ This formulation is particularly well-suited for large-scale global search algorithms as in its simplest form, it can be posed with only box constraints on the variable, making it suitable for metaheuristic-based global optimization algorithms.¹⁰⁻¹² By incorporating the bodies at which the spacecraft is to conduct fly-bys as integer variables to the problem, the MGA-1DSM model can also optimize across various fly-by sequences.

*Equal Contribution, PhD Student, School of Aerospace Engineering, Georgia Institute of Technology, 30332.

†Equal Contribution, PhD Student, Aeronautics & Astronautics, Stanford University, 94305.

Table 1: Parameters of the Trappist-1 tour problem

| Variable | Value |
|---|--------------------------|
| Star μ , km^3/s^2 | 1.19246521e+10 |
| Safety radius r_{crit} , km | 834840 |
| Reference epoch T_{ref} | 2019-Mar-02 00:00:00 UTC |
| Starting radius R_{start} , km | 1,495,978,706.91 |
| Starting speed V_{start} , km/s | 10 |
| Planet radius R_k ($k = 0, \dots, 6$), km | 6378 |

In this work, we introduce the procedures that were developed during SpOC to tackle the tour design problem. Fundamentally, two types of tours are hypothesized, each prioritizing either the minimization of ΔV or TOF. Part of the difficulty in the tour problem is the selection of fly-by sequence, which is combinatorial in nature. To this end, we introduce a metric based on the achievable kinetic energy change of the spacecraft through a given fly-by; this enables us to prioritize the more effective planets to fly-by at the earlier half of the tour, and only vary the sequence of the latter planets, thus reducing the combinatorial dimension. From a method standpoint, we leverage an island-based parallel metaheuristics paradigm^{13–15} along with a sliding local optimization framework for improving triplets of fly-by sequences within the tour, which we refer to as Triplet Boosting. We report on these methods applied to the SpOC problem, which serves as an analog to a possible moon tour problem in the solar system context.

This paper is organized as follows: first, the problem posed in the SpOC, along with the modified MGA-1DSM, are introduced. This is followed by a discussion of the uncovered intuitions into this type of tour, along with heuristics developed to reduce the problem complexity. Thirdly, some of the optimization techniques devised for this problem are introduced. The results found for the Trappist-1 tour are then reported. Finally, concluding remarks are presented.

PROBLEM DESCRIPTION

The problem consists of designing a tour of the seven planets of the Trappist-1 system, minimizing both the total ΔV consumption and TOF. The order in which the planets are to be visited referred to hereafter as the *sequence*, as well as the corresponding trajectory, dictated by variables such as the times of flight and fly-by geometry, is to be determined. This is formulated as a modified MGA-1DSM problem, provided by the organizers. Since the problem has two objectives, the scoring between competitors is done based on a hypervolume with the reference point at $\Delta V = 2500$ m/s and TOF = 4000 days.

Table 1 gives the main parameters of the problem, and Table 2 shows the given information of the Trappist-1 system. It is noteworthy that these planets have very short periods, making the Trappist-1 system comparable to the Jupiterian and Saturnian moon systems.

Overview of Modified MGA-1DSM

The modified MGA-1DSM problem begins the tour at a prescribed distance far from the star as well as its planets, at $R_{\text{start}} = 10$ AU, with a prescribed speed of $V_{\text{start}} = 10^4$ m/s, at a reference epoch T_{ref} . This may be understood as starting the tour at the sphere of influence (SOI) of the Trappist-1 star. The actual starting position vector, $\mathbf{R}_{\text{start}}$, may be chosen anywhere on this sphere

Table 2: Orbital elements at reference epoch of planets in the Trappist-1 system data considered for the SpOC problem

| Index | Body | a , km | e | i , deg | Ω , deg | ω , deg | M , deg | μ , km ³ /s ² |
|-------|------|-----------|----------|-----------|----------------|----------------|-----------|---|
| 0 | b | 1,726,412 | 4.55e-03 | 1.100 | 238.359 | 126.222 | 33.410 | 5.42117394e+05 |
| 1 | c | 2,364,738 | 1.08e-03 | 1.300 | 221.022 | 80.780 | -89.795 | 5.16943538e+05 |
| 2 | d | 3,331,123 | 6.24e-03 | 0.500 | 90.265 | 147.445 | -28.385 | 1.53814412e+05 |
| 3 | e | 4,378,053 | 5.77e-03 | 0.300 | 344.160 | 135.094 | 45.002 | 2.74488719e+05 |
| 4 | f | 5,760,326 | 8.61e-03 | 0.010 | 171.427 | 179.829 | -145.164 | 4.12597189e+05 |
| 5 | g | 7,006,907 | 4.00e-03 | 1.200 | 166.633 | 25.312 | 12.599 | 5.24878452e+05 |
| 6 | h | 9,262,180 | 3.78e-03 | 2.300 | 282.809 | 176.197 | -12.121 | 1.27946920e+05 |

within a latitude of ± 30 deg. Then, the initial leg of the tour is a Lambert-leg, connecting $\mathbf{R}_{\text{start}}$ at the initial epoch t_0 with the position vector of the first planet of the sequence \mathbf{R}_{P_0} with a chosen time of flight TOF_0 . The associated cost with this initial leg is computed by

$$\Delta V_1 = V_1^{L_0} - V_{\text{start}} \quad (1)$$

where $V_k^{L_j}$ with $k \in [1, \dots, 7]$ and $j = [0, 1]$ denotes the velocity magnitude of the k^{th} Lambert leg at the beginning ($j = 0$) or end ($j = 1$) of the leg. Note that this expression assumes the initial velocity vector $\mathbf{V}_{\text{start}}$ can always be aligned with the post-maneuver velocity vector direction. Following the arrival at the first planet, the remaining 6 legs to visit the 6 remaining planets following the original MGA-1DSM formulation. All Lambert legs are assumed to have less than a single revolution and to be counter-clockwise.

The decision vector consists of both continuous variables x_c and integer variables x_i . The integer variables are

$$\mathbf{x}_i = [s_1, s_2, s_3, s_4, s_5, s_6, s_7] \quad (2)$$

which corresponds to the order of the visited planets. The continuous variables are given by

$$\mathbf{x}_c = [\mathbf{x}_{\text{leg } 0}, \mathbf{x}_{\text{leg } 1}, \mathbf{x}_{\text{leg } 2}, \mathbf{x}_{\text{leg } 3}, \mathbf{x}_{\text{leg } 4}, \mathbf{x}_{\text{leg } 5}, \mathbf{x}_{\text{leg } 6}] \quad (3)$$

$$\mathbf{x}_{\text{leg } k} = \begin{cases} [u, v, T_0] & \text{if } k = 0 \\ [\beta_k, \xi_{p,k}, \eta_k, T_k] & \text{if } k = 1, \dots, 6 \end{cases}$$

The starting position vector is constructed from $\mathbf{x}_{\text{leg } 0}$ via

$$\mathbf{R}_{\text{start}} = R_{\text{start}} [\cos \phi \cos \theta \quad \cos \phi \sin \theta \quad \sin \phi]^T \quad (4)$$

where

$$\begin{aligned} \theta &= 2\pi u \\ \phi &= \arccos(2v - 1) - \frac{\pi}{2} \end{aligned} \quad (5)$$

Then, the first Lambert leg is then solved between $\mathbf{R}_{\text{start}}$ and the position vector of the first planet, given by s_1 , at epoch $T_{\text{ref}} + T_0$, with time-of-flight T_0 .

The consecutive MGA-1DSM legs begin with the planetary fly-by at the k^{th} planet with a fly-by radius $r_{p,k}$ given by

$$r_{p,k} = \xi_{p,k} R_k \quad (6)$$

where R_k is the radius of the planet, and a plane-orientation angle β_k , resulting in an outgoing velocity $\mathbf{v}_{k,\text{out}}$ given by¹⁰

$$\begin{aligned}
\mathbf{v}_{k,\infty \text{ in}} &= \mathbf{v}_{k,\text{in}} - \mathbf{V}_{s_k} \\
e_{\text{fly-by}} &= 1 + \frac{r_{p,k} v_{k,\infty \text{ in}}^2}{\mu_{s_k}} \\
\delta_{\text{fly-by}} &= 2 \arcsin(1/e_{\text{fly-by}}) \\
\mathbf{v}_{k,\infty \text{ out}} &= \tilde{v}_{k,\text{in}} \left[\cos(\delta_k) \hat{\mathbf{i}} + \cos(\beta_k) \sin(\delta_k) \hat{\mathbf{j}} + \sin(\beta_k) \cos(\delta_k) \hat{\mathbf{k}} \right] \\
\mathbf{v}_{k,\text{out}} &= \mathbf{V}_{s_k} + \mathbf{v}_{k,\infty \text{ out}}
\end{aligned} \tag{7}$$

where the unit vectors are given by

$$\begin{aligned}
\hat{\mathbf{i}} &= \frac{\tilde{\mathbf{v}}_{k,\text{in}}}{\tilde{v}_{k,\text{in}}} \\
\hat{\mathbf{j}} &= \frac{\hat{\mathbf{i}} \wedge \mathbf{V}_{s_k}}{\|\hat{\mathbf{i}} \wedge \mathbf{V}_{s_k}\|} \\
\hat{\mathbf{k}} &= \hat{\mathbf{i}} \wedge \hat{\mathbf{j}}
\end{aligned} \tag{8}$$

and the incoming velocity $\mathbf{v}_{k,\text{in}}$ is obtained from the previous leg, and \mathbf{V}_{s_k} is the planet's velocity at the encounter epoch; note that the fly-by is assumed to be instantaneous, resulting in a discrete change in spacecraft velocity. During the fly-by, the spacecraft flies on a hyperbolic orbit with eccentricity $e_{\text{fly-by}}$ and turn-angle $\delta_{\text{fly-by}}$. The post-fly-by spacecraft state is propagated by a time of flight $\eta_k T_k$. Finally, a Lambert problem is solved to connect this propagated position with the position of the next planet in the sequence, s_{k+1} , at the next encounter epoch $T_{\text{encounter},k+1}$, given by

$$T_{\text{encounter},k+1} = T_{\text{ref}} + \sum_{i=0}^k T_i \tag{9}$$

with a time of flight $(1 - \eta_k)T_k$. In effect, this results in a velocity discontinuity of magnitude ΔV_k at epoch

$$T_{\text{DSM},k} = T_{\text{ref}} + \sum_{i=0}^{k-1} T_i + \eta_k T_k \tag{10}$$

which corresponds to the DSM of this leg.

Optimization Problem Formulation

The resulting optimization problem is multi-objective mixed-integer nonlinear programming (MO-MINLP), given by

$$\begin{aligned}
\min_{\mathbf{x}_c, \mathbf{x}_i} \quad & \Delta V, \text{ TOF} \\
\text{s.t.} \quad & h_{\text{sequence}}(\mathbf{x}_i) = 0 \\
& g_{\text{periapsis}}(\mathbf{x}_c, \mathbf{x}_i) \leq 0
\end{aligned} \tag{11}$$

where \mathbf{x}_c is the vector of continuous decision variables, and \mathbf{x}_i is the vector of integer decision variables. The objectives are given by

$$\Delta V = \sum_{k=1}^7 \Delta V_k \tag{12}$$

Table 3: Bounds on optimization variables

| Variable | Lower bound value ($\underline{\cdot}$) | Upper bound value ($\bar{\cdot}$) |
|----------|---|-------------------------------------|
| u | 0 | 1 |
| v | 0.25 | 0.75 |
| T_0 | 5 | 2000 |
| β | 0 | 2π |
| r_p | 1.1 | 100 |
| η | 0.001 | 0.999 |
| T_k | 5 | 2000 |

$$\text{TOF} = \sum_{k=1}^7 T_k \quad (13)$$

The equality constraint h_{sequence} is to ensure that each planet is visited exactly once, and is given by

$$h_{\text{sequence}}(\mathbf{x}_i) = |\{\mathbf{x}_i\}| - 7 \quad (14)$$

which requires the set of \mathbf{x}_i to be size 7. The inequality constraint $g_{\text{periapsis}}$ is to ensure the periapsis of the spacecraft trajectory is at all times above a given threshold radius r_{crit} from the star, and is given by

$$g_{\text{periapsis}}(\mathbf{x}_c, \mathbf{x}_i) = r_{\text{crit}} - \min_k(r_{\text{min},k}) \quad (15)$$

where $r_{\text{min},k}$ is the periapsis with respect to the star during the k^{th} leg. Practically speaking, this means that for each leg, the periapsis of the spacecraft immediately after the fly-by and immediately after the DSM must both be checked, and the smaller value is taken as $r_{\text{min},k}$.

If the sequence of the tour is fixed, the problem reduces to multi-objective nonlinear programming (MO-NLP), given by

$$\begin{aligned} \min_{\mathbf{x}_c} \quad & \Delta V, \text{TOF} \\ \text{s.t.} \quad & g_{\text{periapsis}}(\mathbf{x}_c) \leq 0 \end{aligned} \quad (16)$$

By removing the integer variables, a large set of efficient algorithms for MO problems can be applied, hence allowing the identification of optimal, albeit likely local, solutions. If the objective is unified via aggregation, the problem can be further reduced to a single-objective nonlinear programming (NLP),

$$\begin{aligned} \min_{\mathbf{x}_c} \quad & w_1 \Delta V + w_2 \text{TOF} \\ \text{s.t.} \quad & g_{\text{periapsis}}(\mathbf{x}_c, \mathbf{x}_i) \leq 0 \end{aligned} \quad (17)$$

Of course, the weight parameters can be expressed as a single scalar ratio w_1/w_2 as well.

PROBLEM ANALYSIS

Insights have been found through a preliminary analysis of the problem. In this section, we comment on the observations made on the decision variables, the overarching types of tour that we aim to find, and the fly-by sequences.

Bounds on Decision Variables

While bounds are provided on the decision variables of the problems, tighter bounds around ranges of known optimal values would help metaheuristic algorithms reach competitive solutions faster. This must be done with caution in order to avoid restricting the search in unfavorable ways, where potentially good solutions would be omitted.

Tour Orbital Plane and Variable v Changing the orbital plane either through fly-by's or DSMs would involve using the effect of the fly-by or the ΔV maneuver itself to this end; it is possible to reason that the least amount of orbital plane change should be conducted. Therefore, it is possible to bound v further than the provided bounds in Table 3, such that the initial inclination of the spacecraft is below the largest inclination among the Trappist-1 system planets. This corresponds to an inclination of 2.3° of planet h, resulting in bounds on $v \in [0.48, 0.52]$.

Bounds on Time of Flight of Initial Leg From the reasons mentioned later in Section., the initial encounter planet is fixed to the inner planet b . Figure.1 shows the relationship between the initial leg TOF and required ΔV sampled in the prescribed bound $v \in [0.48, 0.52]$. The required ΔV of the initial leg is determined exclusively by the TOF, due to the geometry of the initial arc. Let r_1, r_2 be the initial and final points of the initial arc. From Lagrange's equation, the time of flight is a function of semi-major axis a , the sum of the distances of the initial and final point from the primary attractor $r_1 + r_2$, and the length of the chord c of the triangle having r_1, r_2 as sides, as follows.

$$\text{TOF} = f(a, r_1 + r_2, c) \quad (18)$$

In the given problem setting, we have $r_1 \gg r_2$, and therefore $r_1 \approx c, r_1 + r_2$. r_1 is also fixed to 10AU from the problem constraints, so $r_1 + r_2$ and c are nearly constant in our problem. Therefore, from Eq.(18), the semi-major axis a and the energy of the initial arc is determined almost exclusively by the TOF. The ΔV is determined by the difference in the (fixed) initial energy and the energy of the initial leg, so it is also determined exclusively by the TOF. From Figure.1, we can see the total ΔV hits the minimum around the total TOF of 1500 days which creates a trade-off between TOF and ΔV when $\text{TOF} \leq 1500$. Considering the orbital period of the first encounter planet is 1.25 days, the phasing of the trajectory could be adjusted by slightly changing the initial TOF, and therefore there is nearly no merit in setting the initial TOF substantially larger than 1500 days. From this observation, we bounded the initial TOF to $\text{TOF} \in [1350, 1500]$.

Types of Tour

The dual objective nature of the tour design problem may be understood as a management task of the spacecraft's energy and mission duration. Two possible philosophies have been hypothesized for the tour, namely (a) one that aims at reducing the energy continuously throughout the tour, and (b) another that initially reduces the energy and then sustains a similar energy level over the remaining time.

The first type referred to hereafter as the *Oberth-type*, is the strategy of choice to prioritize the minimization of the TOF. This is achieved by continuously pumping energy out of the spacecraft, both through fly-by's and maneuvers. The maneuvers should be placed near the periapsis of each leg in order to leverage the Oberth effect. This type of tour would result in relatively low apoapsis, which makes the adjustment of the orbital planes for successive planetary encounters more costly.

The second type, on the other hand, referred to hereafter as the *VILT-type*, is the strategy of choice to prioritize the minimization of the ΔV . The idea behind this type of tour is similar to V-infinity

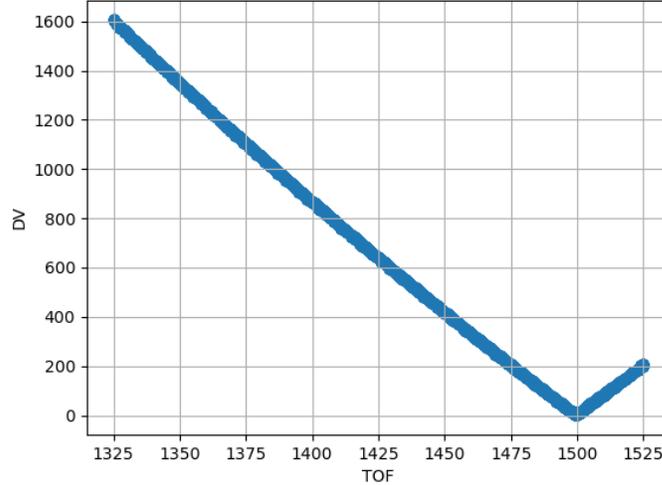


Figure 1: Time of flight (TOF) vs required Delta-V for the initial leg. For each TOF, the parameter u is sampled from 51 values in the range $[0, 1]$, and v is sampled from 5 values in the range $[0.48, 0.52]$

leveraging transfers (VILTs), where only minor corrective maneuvers are placed near the apoapsis in order to adjust the orbital plane and/or phase for the next planetary encounter. In this type of tour, the reduction of the spacecraft energy is less aggressive, hence the apoapsis remains higher than the Oberth-type. While this causes the TOF to be longer, the higher apoapsis is beneficial for keeping the orbital plane adjustment cost low.

Figure 2 shows the time-history of the spacecraft energy of the solutions found for the Trappist-1 tour problem. The wide variation in energy over time illustrates the existence of the Oberth-type tours, VILT-type tours, as well as concessions of the two types.

Fly-by Sequence Considerations

For both types of tours, conducting a fly-by that provides a bigger change in energy sooner is preferable, as it enables the tour to profit from the lowered energy for a longer duration. Looking at Table 2 it is possible to observe that the planets have values of μ_k that vary by up to about 4 fold. In particular, planets b, g, and c have the largest μ_k in descending order. The effectiveness of a fly-by is however not only determined by the value of μ_k of the planet, but also by the distance of the planet from its host star, as well as the closest fly-by radius at which the spacecraft can fly. Furthermore, due to the need for phasing, it is noted that the sequence corresponding to decreasing fly-by efficiency is not necessarily the optimal sequence.

Metric of fly-by efficiency The efficiency of the fly-by comes from the ability of a planet to “bend” the path of a spacecraft; in effect, this is embodied by the eccentricity of the fly-by hyperbola within the sphere of influence of the flyby body, which we seek to bring as close as possible to 1. Note that a decrease in eccentricity also results in a longer time of flight within the sphere of influence of the planet, but this is still assumed to have a negligible effect on the total time of flight of the tour due to the high energies associated with the spacecraft as it conducts fly-by with respect

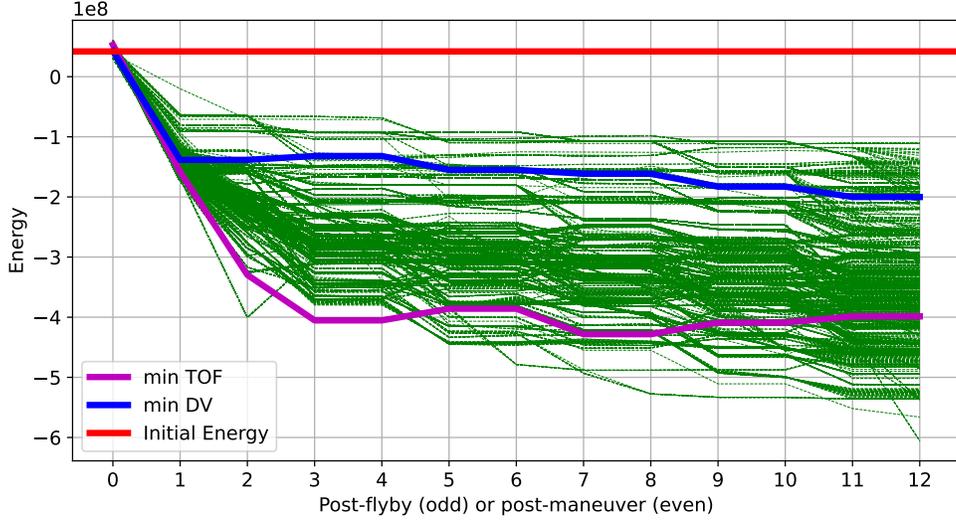


Figure 2: History of energy after each fly-by and maneuver for the Trappist-1 tour problem.

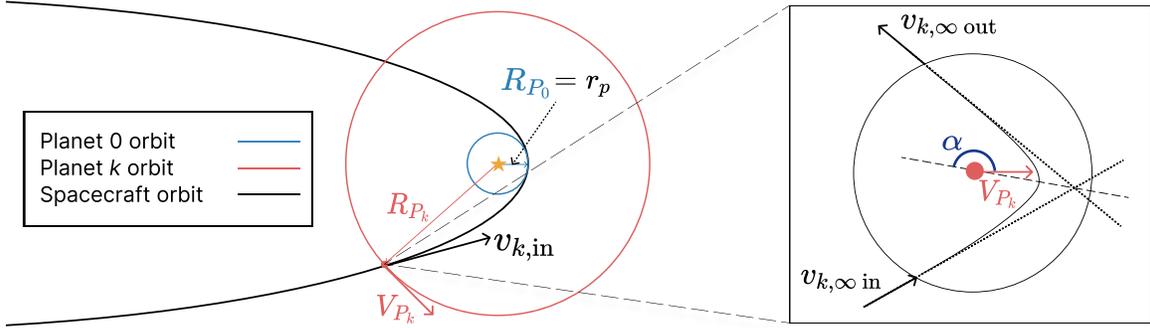


Figure 3: Simplified fly-by analysis for computing the maximum change in kinetic energy obtainable from a fly-by with the k^{th} planet.

to the planets.

To arrive at a heuristic metric for evaluating the efficiency of conducting a fly-by at a given planet, we consider fly-by's of the spacecraft on an elliptical orbit whose periapsis corresponds to the semi-major axis of the innermost planet R_{P_0} , as illustrated in Figure 3.

Assuming all planets are on circular orbits and are coplanar with the spacecraft's ellipse, for a given apoapsis r_A , the interplanetary velocity magnitude $v_{k,\text{in}}$ and flight path angle γ of the spacecraft prior to its fly-by at the k^{th} planet, where $k \in \{1, \dots, 7\}$, are obtained from the Keplerian relations

$$v_{k,\text{in}} = \sqrt{\mu \left(\frac{2}{R_{P_k}} - \frac{1}{r_A + R_{P_k}} \right)} \quad (19)$$

where R_{P_k} represents the semi-major axis of planet k , and

$$\gamma = \arctan \left(\frac{e \sin f}{1 + e \cos f} \right) \quad (20)$$

where e and f are respectively the eccentricity and true anomaly of the interplanetary trajectory of the spacecraft prior to fly-by, given by

$$e = \frac{r_A - r_P}{r_A + r_P} = \frac{r_A - R_{P_0}}{r_A + R_{P_0}} \quad (21)$$

$$\cos f = \frac{1}{e} \left(\frac{a(1 - e^2)}{R_{P_k}} - 1 \right) \quad (22)$$

Note again that the periapsis of the spacecraft's interplanetary trajectory r_P is assumed to coincide with the semi-major axis of the inner-most planet rather than the fly-by planet of interest. Using the interplanetary velocity magnitude $v_{k,\text{in}}$ and the flight path angle γ , the V-infinity magnitude of the spacecraft in the frame of reference of planet k is given by

$$v_{k,\infty\text{in}} = \sqrt{(v_{k,\text{in}} \cos \gamma - V_{P_k})^2 + (v_{k,\text{in}} \sin \gamma)^2} \quad (23)$$

where $V_{P_k} = \sqrt{\mu/R_{P_k}}$ is the velocity magnitude of planet k , for which a circular orbit has been assumed. Then, from basic analyses of velocity diagrams, the change in kinetic energy of the spacecraft in the interplanetary frame of reference is given by¹⁶

$$\Delta E_k = \frac{2V_{P_k} v_{k,\infty\text{in}} \cos \alpha}{1 + \frac{r_{p,k} v_{k,\infty\text{in}}^2}{\mu_k}} \quad (24)$$

where α is the angle between the planet's velocity vector and the vector difference $\mathbf{v}_{k,\infty\text{out}} - \mathbf{v}_{k,\infty\text{in}}$. Since the aim of the fly-by in the context of this work is to reduce the energy of the spacecraft, the minimum value of ΔE_k can be obtained when $\alpha = \pi$, and the fly-by radius $r_{p,k}$ is set at its lowest possible value.

Figure 4 shows values of ΔE_k for all seven planets $k \in \{1, \dots, 7\}$ with varying values of the spacecraft's interplanetary apoapsis r_A . Effectively, by visiting the planets in decreasing order of $|\Delta E_k|$, we are achieving the theoretically optimal sequence of fly-by from the sole point of view of reducing the energy of the spacecraft as early in the tour as possible.

Down Selection of Fly-by Sequence Based on the provided information on the planets, the innermost planet provides by far the largest kinetic energy change. Since the first leg can be started from any location at 10 AU, phasing does not become an issue either. Also, for this initial fly-by, in particular, both the Oberth-type and VILT-type tours need to reduce the spacecraft energy. Hence, planet 0 makes for a good candidate as the first fly-by planet.

Subsequent planets may also be fixed following a similar argument, however, phasing constraints make this choice more nuanced. As a strategy, we initially only froze the first planet and permuted the remaining 6, but eventually reduced the permutation of to be only for the last 4 planets, fixing the first three planets to the sequence $[0, 1, 5]$, following a decreasing order in ΔE_k .

OPTIMIZATION TECHNIQUES

In an attempt to obtain a diverse Pareto front, the tour problem is solved by fixing the fly-by sequence based on the considerations previously introduced and applying various techniques to further improve the search process. This is repeated for all combinations of promising sequences. The initial solution is sought through the use of metaheuristic algorithms organized in the so-called

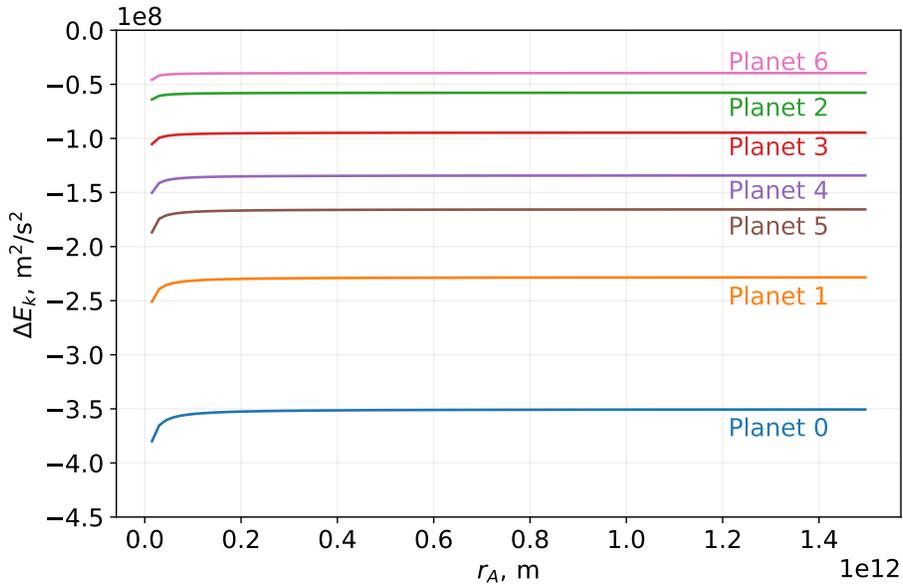


Figure 4: Values of ΔE_k for increasing apoapsis radius r_A . The optimal energy reduction sequence via fly-by alone is [0, 1, 5, 4, 3, 2, 6].

archipelago paradigm,¹⁵ where different algorithms can evolve independently in parallel while also sharing solutions at certain intervals.

Due to the difficulty of phasing and encountering planets at the right timings, it has been found that improving the TOF of a given solution is extremely challenging. In contrast, the ΔV could be reduced with relative ease through small adjustments of the fly-by parameters or DSM location. We refer to this latter process as “boosting” a given solution. Specifically, we introduce a sliding algorithm that takes triplets of planets at a time and optimizes the two arcs between them, which we call *triplet boosting*.

In this section, we first provide a brief discussion on the application of metaheuristic techniques relevant to generating promising solutions. Then, we introduce the triplet boosting scheme.

Overview of Metaheuristics Techniques

Extensive use of the pygmo library¹⁷ has been made to use the archipelago architecture with a combination of various metaheuristic algorithms. The archipelago is particularly useful as it allows for different algorithms to be dispatched at once when there is no prior insight as to which algorithm is best suited for the problem at hand. In fact, Izzo et al¹⁵ report performance improvements in many test problems when the “migration” operation, where the best solution obtained from one algorithm is inserted in the solution pool of another algorithm, is allowed.

As discussed in the Problem Description Section, the original problem is a constrained, MO-MINLP, for which no effective algorithm exists. Typically in the context of using metaheuristics, violation of constraints may be added to the objective function as large penalty terms, thus converting the problem to be unconstrained. Even then, unconstrained MO-MINLP remains difficult to solve. We have performed a few experiments on a MO-MINLP formulation using the Non-Dominated Sorting GA (NSGA2)¹⁸ and the Multi-objective Hypervolume-based ACO (MHACO),¹⁹ but soon

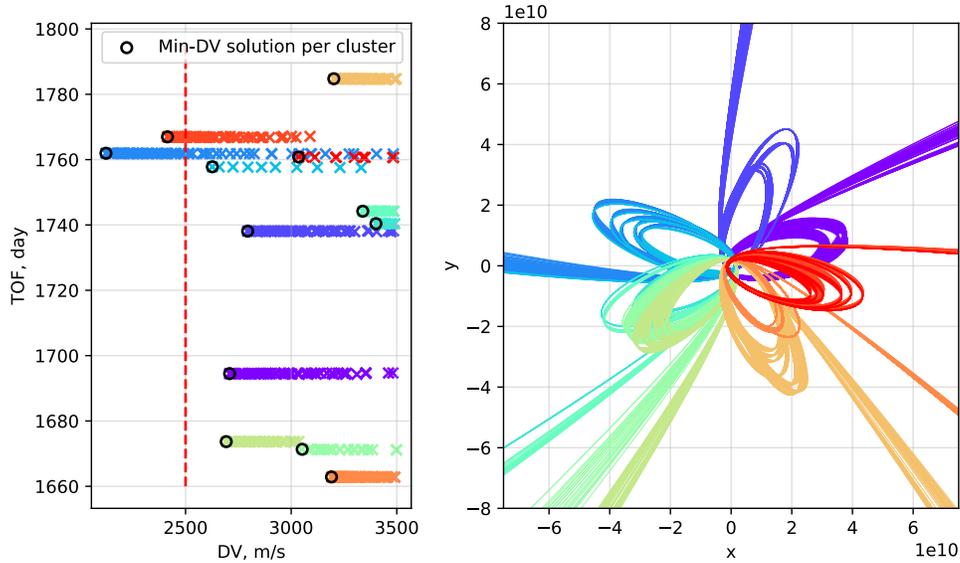


Figure 5: DBSCAN of pre-boosted solutions in terms of leg times of flight.

found that solving multiple continuous MO-NLPs, with fixed sequences at each instance, yields better results.

MO-NLPs may also be solved with NSGA2 and MHACO, or also in addition the Multi-objective EA with Decomposition (MOEA/D).²⁰ However, we note that due to the aforementioned significant difficulty in improving the TOF compared to improving the ΔV , we found that aggregating the two objectives and using algorithms for NLPs is best suited to this problem. To solve the NLP, the Self-adaptive Differential Evolution (saDE)²¹ and the Extended Ant Colony Optimization (GACO)¹⁹ are used extensively.

Boosting solutions

Due to the nature of many-fly-by trajectory, finding the right timings of encountering each celestial body is the major challenge; as such, once a solution is found, it has been found to be difficult to obtain meaningful improvement on the time of flight, as the phasing of the planets are “locked-in”. In contrast, improvements in ΔV have been far more likely to be possible, often by relocating and fine-tuning the timing and direction of the maneuvers. Hence, a process internally referred to as “boosting” has been devised to improve promising solutions, particularly in terms of the total ΔV .

DBSCAN in terms of Time of Flight In order to categorize the trajectories into families, DBSCAN in terms of times of flight of each leg has proven to be particularly useful at automatically identifying families of trajectories. DBSCAN is a density-based data clustering algorithm first introduced by Ester et al.²²

Figure 5 shows an example of DBSCAN applied to a set of solutions with a fixed sequence. The clustered solutions correspond to families of trajectories sharing similar directions of periapsis. The best solution with respect to the ΔV from each cluster can then easily be identified as a candidate for further improvement.

Triplet Boosting We performed “triplet boosting” to update the timing of the deep space maneuvers and planetary flybys in an iterative manner to reduce ΔV locally while keeping the sequence fixed and planet flyby and phasing constraints satisfied. Here, a “triplet” is defined as a set of trajectories that connects three planets, containing two deep-space maneuvers and an intermediate flyby, as shown in Fig.6 and Fig.7.

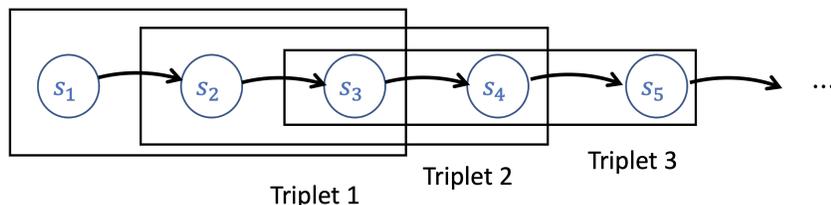


Figure 6: Triplets of flyby planets. Five triplets are constructed from the seven-planet sequence.

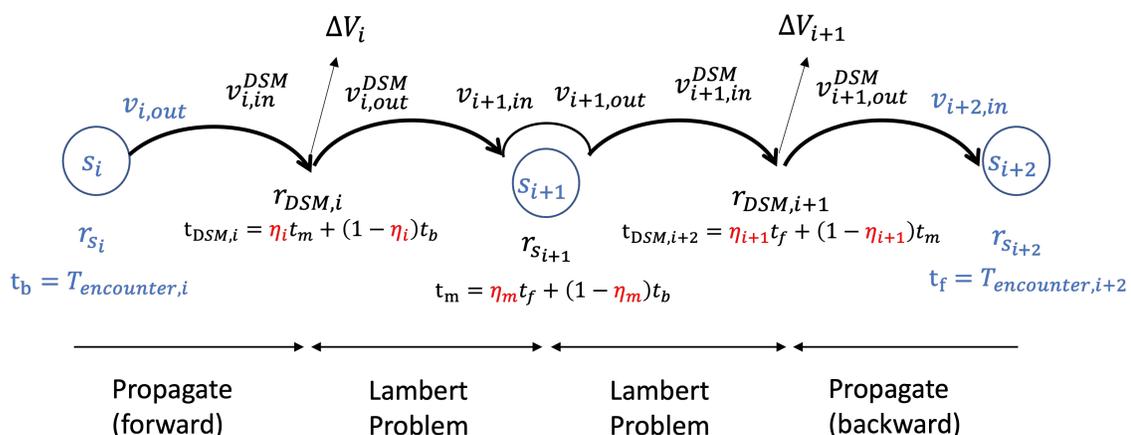


Figure 7: Parameters of each triplet. The three variables colored in red are the variables that are optimized and updated. The nine parameters that are colored in blue are the parameters that are fixed during optimization (boundary conditions). Other parameters in black are the intermediate or output variables. In the “Triplet Boosting” optimization, the first and fourth leg is constructed from forward and back propagation from the boundary conditions, and the two middle legs are calculated by solving the Lambert Problem.

The high-level procedure is described in Algorithm.1. For each triplet, we optimize two parameters that determine the timing of the deep space maneuver (η_i, η_{i+1}) and the flyby time of the second planet in the triplet (η_m) to minimize the ΔV for the two deep space maneuvers. During the optimization, we constrain the initial and final conditions (position, velocity, time) of the triplet so that the updated variables will not change the flyby trajectories outside the triplet.

The optimization problem which is solved for triplet i is described as follows.

$$\begin{aligned} & \min_{\eta_i, \eta_{i+1}, \eta_m} \Delta V_i + \Delta V_{i+1} \\ & s.t. \begin{cases} \xi_{i+1} \geq 1.1 \\ |\tilde{\mathbf{v}}_{i+1,in}| - |\tilde{\mathbf{v}}_{i+1,out}| = 0 \\ 0.001 \leq \eta_i, \eta_{i+1}, \eta_m \leq 0.999 \end{cases} \end{aligned} \quad (25)$$

Above, $\Delta V_i, \Delta V_{i+1}, \xi_{i+1}, \tilde{\mathbf{v}}_{i+1,in}, \tilde{\mathbf{v}}_{i+1,out}$ (see Fig.7 and ‘‘Problem Description’’ section for the details) are functions of the optimization variable $\eta_i, \eta_{i+1}, \eta_m$ and the boundary constraints $\mathbf{r}_{s_i}, \mathbf{v}_{i,out}, \mathbf{r}_{s_{i+2}}, \mathbf{v}_{i+2,in}, T_{\text{encounter},i}, T_{\text{encounter},i+2}$. The procedure of this calculation is shown in Algorithm 2. Optimization is performed via Sequential Least Squares Programming (SLSQP) method in the `scipy.optimize` function,²³ and the initial guesses of the three η variables are calculated from the solutions before boosting.

Algorithm 1 High level algorithm of triplet boosting

- 1: **Given:** $\mathbf{x}_i, \mathbf{x}_c$ that satisfies constraint
 - 2: Propagate trajectory and obtain $T_{\text{encounter},j}, \mathbf{r}_{s_j}, \mathbf{v}_{j,in}, \mathbf{v}_{j,out}$ for $j = 1, \dots, 7$
 - 3: **for** $k = 1 : N_{\text{iter}}$ **do** ▷ Iterate several times
 - 4: **for** $i = 1 : 5$ **do** ▷ For each triplet
 - 5: $\eta_i, \eta_{i+1}, t_m \leftarrow$ Solve Optimization Problem (25) ▷ Objective and constraint are calculated using Algorithm 2
 - 6: Update $\eta_{i+1}, T_{\text{encounter},i+1}, \mathbf{r}_{s_{i+1}}, \mathbf{v}_{i+1,in}, \mathbf{v}_{i+1,out}$
-

RESULT ON TRAPPIST-1 SYSTEM

The trajectory search pipeline presented thus far has been applied for the Trappist-1 System tour problem. This section reports on the Pareto front that has been found.

Figure 8 shows the 1000 Pareto front solutions with colors based on the sequences, and Figure 9 shows the same Pareto front solutions but colored in terms of values of specific design variables. Firstly, it is possible to note that all sequences belonging to the Pareto start with the innermost planet (planet 0), and 7 out of 8 of the best solutions’ sequences then transfer to planet 1. As for the third planet, 5 out of 8 solutions utilize planet 5. This aligns with the order of the flyby-efficiency shown in Figure.4.

Visualizing Selected transfers

From the Pareto front in Figure 8, the 8 circled solutions are selected and studied in further detail. The trajectories are shown in Figure 10. There exists a difference in the DSM placement strategy of the first two arcs among families: the fast families (e.g. sequence 0154362) conduct DSM near the periapsis to reduce orbital energy, while families with smaller ΔV s conduct the DSM near the apoapsis from the beginning (e.g. sequence 0243651), to achieve necessary plane breaking with minimum effort. However, after the third flyby, every family performs ΔV near the apoapsis to keep the total ΔV within the 2500 m/s upper bound constraint. From Figure 8 we also observe a strong relationship between the initial arc TOF and the total ΔV : almost all trajectories with total ΔV below 1500 m/s have the initial TOF around 1500 days, which is energy effective as shown in Figure 1.

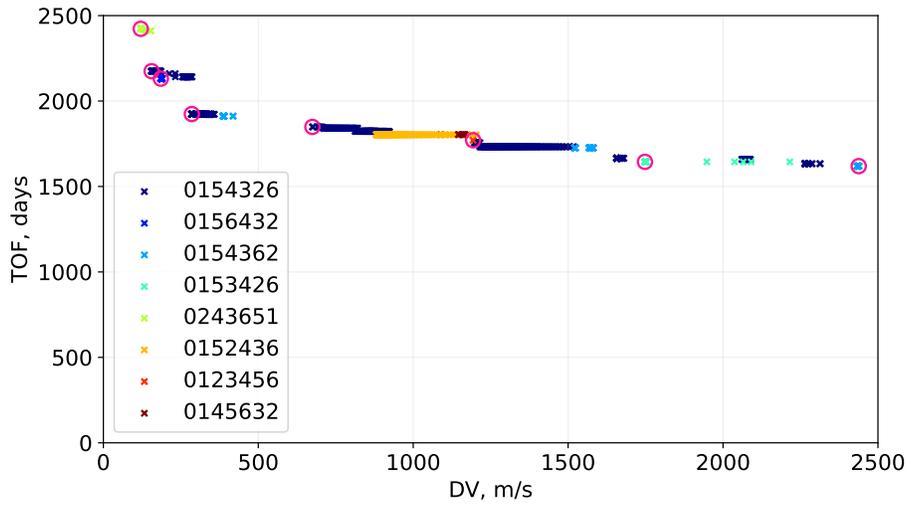


Figure 8: Best 1000 Pareto front solutions, colored by fly-by sequence. Black circles indicated selected solutions for further analysis.

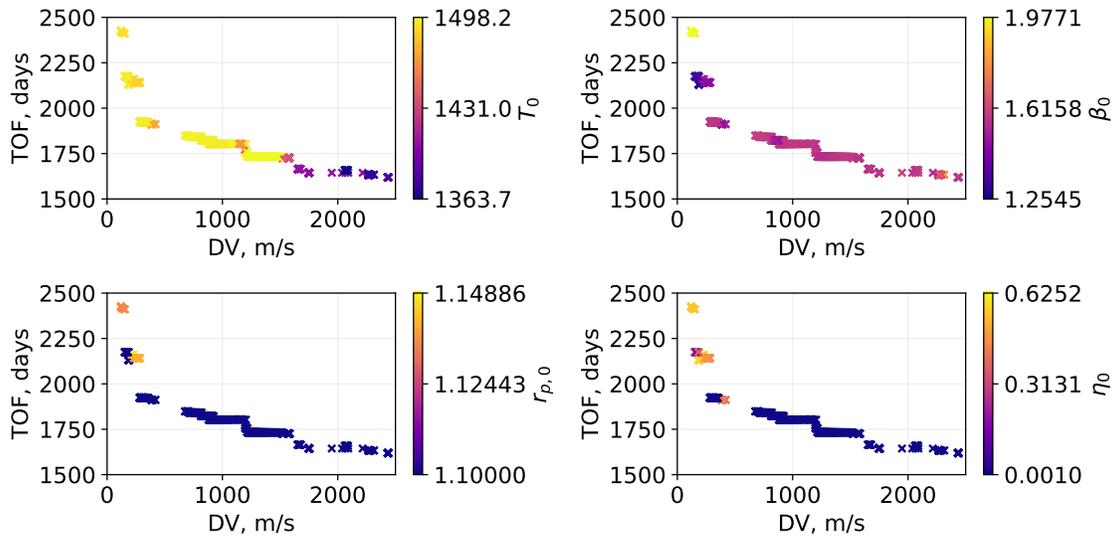


Figure 9: Best 1000 Pareto front solutions, colored by selected decision variable values.

Algorithm 2 Computing the objective and constraint function of triplet boosting for triplet i

- 1: **Given:** $\mathbf{r}_{s_i}, \mathbf{v}_{i,out}, \mathbf{r}_{s_{i+2}}, \mathbf{v}_{i+2,in}, T_{\text{encounter},i}, T_{\text{encounter},i+2}$ ▷ See Fig.7 for definitions
 - 2: **Variable:** $\eta_i, \eta_{i+1}, \eta_m$ ▷ Variables to optimize
 - 3: $t_b \leftarrow T_{\text{encounter},i}, \quad t_f \leftarrow T_{\text{encounter},i+2}$
 - 4: $t_m \leftarrow t_b + \eta_m(t_f - t_b)$
 - 5: $\mathbf{r}_{s_{i+1}}, \mathbf{v}_{s_{i+1}} \leftarrow \text{PlanetEphemeris}(\text{oe}(s_{i+1}), t_m)$
 - 6: $\mathbf{r}_{DSM,i}, \mathbf{v}_{i,in}^{DSM} \leftarrow \text{Propagate}(\mathbf{r}_{s_i}, \mathbf{v}_{i,out}, \eta_i(t_m - t_b), \mu)$
 - 7: $\mathbf{r}_{DSM,i+1}, \mathbf{v}_{i+1,out}^{DSM} \leftarrow \text{Propagate}(\mathbf{r}_{s_{i+2}}, \mathbf{v}_{i+2,in}, -(1 - \eta_{i+1})(t_f - t_m), \mu)$ ▷ Propagate backwards in time
 - 8: $\mathbf{v}_{i,out}^{DSM}, \mathbf{v}_{i+1,in} \leftarrow \text{SolveLambert}(\mathbf{r}_{DSM,i}, \mathbf{r}_{s_{i+1}}, (1 - \eta_i)(t_m - t_b), \mu_c)$ ▷ Lambert Problem
 - 9: $\mathbf{v}_{i+1,out}, \mathbf{v}_{i+1,in}^{DSM} \leftarrow \text{SolveLambert}(\mathbf{r}_{s_{i+1}}, \mathbf{r}_{DSM,i+1}, \eta_{i+1}(t_f - t_m), \mu_c)$
 - 10: $\Delta V_i \leftarrow |v_{DV1}^+ - v_{DV1}^-|$
 - 11: $\Delta V_{i+1} \leftarrow |v_{DV2}^+ - v_{DV2}^-|$
 - 12: **Objective:** $\Delta V_i + \Delta V_{i+1}$ ▷ Objective: Total ΔV
 - 13: $\tilde{\mathbf{v}}_{i+1,in} = \mathbf{v}_{i+1,in} - \mathbf{v}_{s_{i+1}}$ ▷ Pre-flyby V_∞
 - 14: $\tilde{\mathbf{v}}_{i+1,out} = \mathbf{v}_{i+1,out} - \mathbf{v}_{s_{i+1}}$ ▷ Post-flyby V_∞
 - 15: $\delta_{fb} = \arccos\left(\frac{\tilde{\mathbf{v}}_{i+1,in} \tilde{\mathbf{v}}_{i+1,out}}{|\tilde{\mathbf{v}}_{i+1,in}| |\tilde{\mathbf{v}}_{i+1,out}|}\right)$ ▷ Compute deflection angle
 - 16: $r_{p,i+1} = \frac{\mu_{s_{i+1}}}{|\tilde{\mathbf{v}}_{i+1,in}|^2} \left(\frac{1}{\sin(0.5\delta_{fb})} - 1 \right)$
 - 17: $\xi_{i+1} = \frac{r_{p,i+1}}{R_{i+1}}$
 - 18: **Constraint1:** $\xi_{i+1} \geq 1.1$ ▷ Flyby radius constraint
 - 19: **Constraint2:** $|\tilde{\mathbf{v}}_{i+1,in}| - |\tilde{\mathbf{v}}_{i+1,out}| = 0$ ▷ Ballistic flyby constraint
-

CONCLUSIONS

In this paper, findings from solving the dual-objective high-energy tour design have been reported. While the problem is posed as a variant of the MGA-1DSM problem, the trajectory has a distinct character from interplanetary fly-by MGA-1DSM trajectories due to the strong constraint of the phasing between intermediate fly-by's that is typical in a high-energy tour.

Both preliminary analysis on this class of problem that is based on analytical orbital mechanics, as well as practical techniques for facilitating the multiobjective global optimization, has been discussed. Notably, two types of transfers, coined as the Oberth-type and VILT-type, have been hypothesized; then, expressions for arriving at the optimal fly-by sequence based on the obtainable maximum change in kinetic energy have been derived. At the optimization stage, a combination of archipelago paradigms for large-scale global optimization problems, coupled with a DBSCAN process for pruning solutions and a boosting strategy that further optimizes the phasing between the subsequent fly-by's, has been proposed.

While some of the design decisions that were made, such as bounds on design variables, were restricted by the problem definition of SpOC, the intuitions and approaches studied in this paper are applicable to tour design problems outside of the competition.

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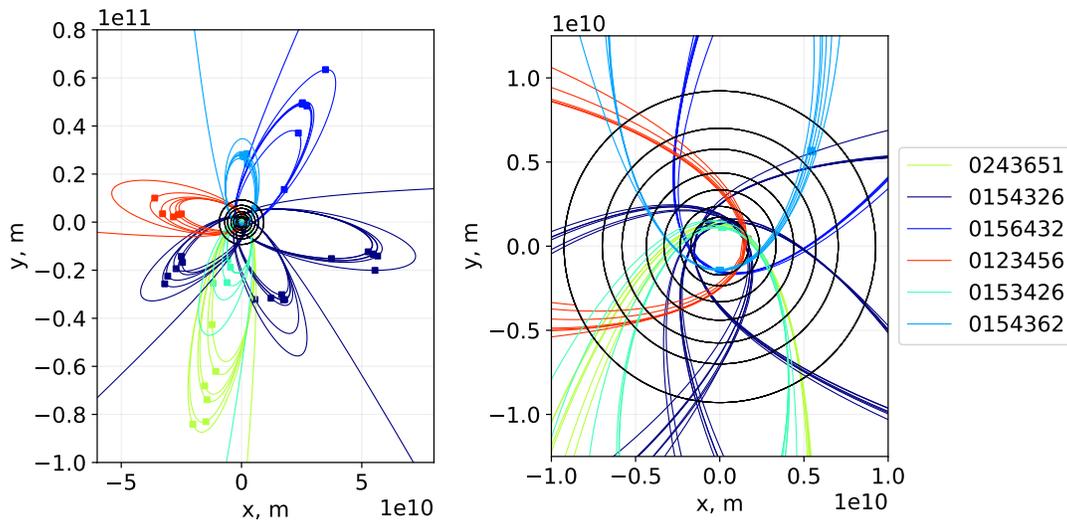


Figure 10: Selected trajectories from Pareto front. Markers indicate DSM locations. Sequence [0, 1, 5, 4, 3, 6, 2] is the fastest solution, and sequence [0, 2, 4, 3, 6, 5, 1] is the minimum ΔV solution.

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