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RELATIVE NAVIGATION FOR SATELLITES IN CLOSE PROXIMITY USING ANGLES-ONLY OBSERVATIONS

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"Relative navigation using angles-only observations is explored in this research. Previous work has shown that the unique relative orbit of a deputy satellite cannot be found using angles-only camera measurements from the chief satellite when a linear model of relative motion is used, due to a lack of observability. This work examines the possibility of partial observability in this case, which consists of a basis vector that corresponds to a family of relative orbits. A Preliminary Orbit Determination (POD) method is introduced that uses 3 Line-Of-Sight (LOS) measurements and provides an initial guess for the basis vector. This guess is differentially corrected with a batch filter that takes in a full set of LOS measurements to hone in on a converged solution for the basis vector. The application of an Extended Kalman Filter (EKF) to this problem is also explored."

INTRODUCTION

In performing relative navigation for satellites in close proximity, two methods of observation are typically used. These are cameras and range sensors on the satellites. The problem that is examined here is a case where the only orbit determination tools are cameras. This means the only information known is the angles one satellite exhibits in the frame of the other, usually represented as a Line-Of-Sight (LOS) unit vector. Woffinden and Geller[1] stated that the unique relative orbit cannot be found using angles-only measurements when a linear model of relative motion is used to model the dynamics and there are no thrusting maneuvers, due to lack of observability. They showed that a family of trajectories whose state histories are proportional to one another (i.e. differ by only a constant scalar multiple) will possess a common LOS history. Figure 1 shows what is meant by a family of relative orbits. This implies that while the unique relative state cannot be determined, it should be possible to determine a basis vector for the trajectory. This can be thought of as the direction of the state vector, whereas its magnitude is unobservable. The goal of this paper is to develop a technique for determining the basis vector of a relative trajectory given angles-only measurements.

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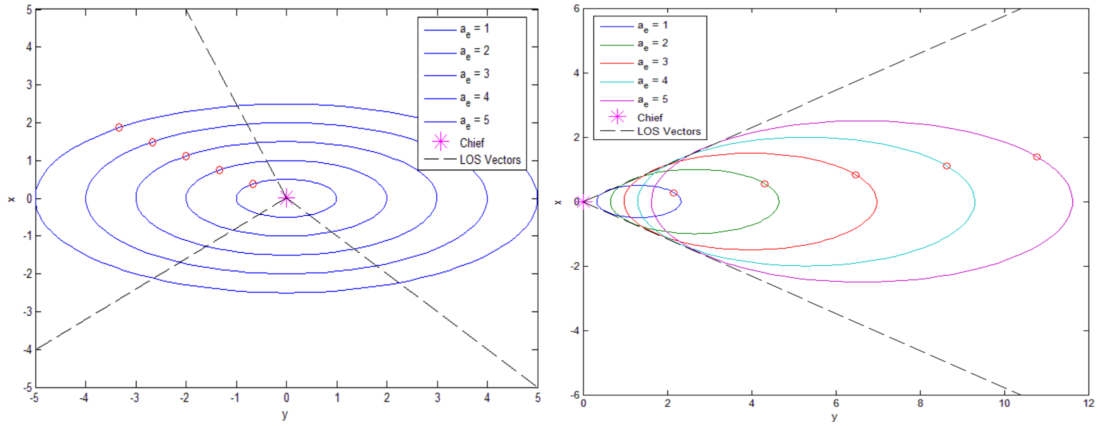

Figure 1. Examples of Families of Relative Orbits

PRELIMINARY RELATIVE ORBIT DETERMINATION METHOD

Initially, a mathematically rigorous Preliminary Orbit Determination (POD) procedure for finding the family of relative orbits that correspond to three angle measurements, which were taken as 3-D LOS unit vectors, was developed. The Hills-Clohessy-Wiltshire (HCW) equations[2,3] were used to model the relative dynamics. Key assumptions in the HCW equations are that the two spacecraft are in close proximity, the chief's orbit has zero eccentricity (and the deputy has zero or a small value of eccentricity), and no perturbations are modeled, just two-body gravitational dynamics. In the HCW equations, the state variables at one time are related to the state variables at another time using a state transition matrix (STM) as follows.

$$\begin{bmatrix} \vec{r}_2 \\ \vec{v}_2 \end{bmatrix} = [\Phi(t_1, t_2)] \begin{bmatrix} \vec{r}_1 \\ \vec{v}_1 \end{bmatrix} \tag{1}$$

Since the LOS unit vectors at each measurement time are known, Eq. (1) can be rewritten as

$$\begin{bmatrix} \vec{r} \\ \vec{v} \end{bmatrix} = [H] \begin{bmatrix} \alpha r \\ \alpha \vec{v} \end{bmatrix} = \begin{bmatrix} \hat{l}_{3X1} & \underline{\underline{0}}_{3X3} \\ \underline{\underline{0}}_{3X1} & \underline{\underline{I}}_{3X3} \end{bmatrix} \begin{bmatrix} \alpha r \\ \alpha \vec{v} \end{bmatrix} \tag{2}$$

where $\hat{l}_{3X1}$ is the LOS unit vector and α is a scaling factor to show that the solution found later is not unique. Therefore, plugging Eq. (2) into Eq. (1) gives

$$[H_2] \begin{bmatrix} \alpha_2 r_2 \\ \alpha_2 \vec{v}_2 \end{bmatrix} = [\Phi(t_1, t_2)][H_1] \begin{bmatrix} \alpha_1 r_1 \\ \alpha_1 \vec{v}_1 \end{bmatrix} \tag{3}$$

The same derivation can be performed to relate the states from time 1 to time 3 as follows

$$[H_3]\begin{bmatrix}\alpha_3 r_3\\ \alpha_3 \vec{v}_3\end{bmatrix} = [\Phi(t_1,t_3)][H_1]\begin{bmatrix}\alpha_1 r_1\\ \alpha_1 \vec{v}_1\end{bmatrix} \tag{4}$$

Setting the previous two equations equal to zero and putting them into matrix form gives

$$\begin{bmatrix}\Phi(t_1,t_2)H_1 & -H_2 & \underline{\underline{0}}_{6X4}\\ \Phi(t_1,t_3)H_1 & \underline{\underline{0}}_{6X4} & -H_3\end{bmatrix}\begin{bmatrix}\alpha_1 r_1\\ \alpha_1 \vec{v}_1\\ \alpha_2 r_2\\ \alpha_2 \vec{v}_2\\ \alpha_3 r_3\\ \alpha_3 \vec{v}_3\end{bmatrix} = \left[\underline{\underline{0}}_{12X1}\right] \tag{5}$$

Solving for the null space of the first matrix, which is 12X12, gives the 12X1 vector of unknown values desired in the second matrix, up to a proportional factor. If the first matrix is full rank, there is no vector of unknowns that satisfies this relationship. For most arbitrary sets of three LOS unit vectors, this will be the case. However, for LOS unit vectors that are all part of the same trajectory, the matrix will be rank deficient by one, therefore there will indeed be a null vector.

This procedure was tested numerically in MATLAB by plugging in a random set of initial local vertical, local horizontal (LVLH) states then propagating the state vector to two later times using the HCW equations. Then the unit directions of the position vectors of these states were put into the POD method. Once the range magnitudes were solved for, they were multiplied into the LOS unit vectors to give the final position state values. It was found that these values were all proportionally equal to the initial position states by the same factor. Thus, the method produced a valid basis vector for the motion. This was verified for several example cases.

Results for POD Method

The initial state values were prescribed as

$$[x_1 \quad y_1 \quad z_1 \quad \dot{x}_1 \quad \dot{y}_1 \quad \dot{z}_1]^T = [-5 \quad 3 \quad 1 \quad 1 \quad 2 \quad 3]^T \tag{6}$$

This state was propagated forward 50 seconds using the HCW equations to give

$$[x_2 \quad y_2 \quad z_2 \quad \dot{x}_2 \quad \dot{y}_2 \quad \dot{z}_2]^T = [50.831 \quad 99.827 \quad 150.91 \quad 1.233 \quad 1.869 \quad 2.995]^T \tag{7}$$

Again, the initial state was propagated forward 100 seconds to give the following values at time 3.

$$[x_3 \quad y_3 \quad z_3 \quad \dot{x}_3 \quad \dot{y}_3 \quad \dot{z}_3]^T = [118.19 \quad 189.4 \quad 300.3 \quad 1.4609 \quad 1.7099 \quad 2.979]^T \tag{8}$$

The position vectors from these states were divided by their magnitudes to give their corresponding LOS unit vectors as

$$\hat{l}_1 = [-0.8452 \quad 0.5071 \quad 0.1690]^T$$
$$\hat{l}_2 = [0.2705 \quad 0.5312 \quad 0.8030]^T \tag{9}$$
$$\hat{l}_3 = [0.3159 \quad 0.5062 \quad 0.8025]^T$$

Plugging these values into the POD method produces a 12X12 matrix with the null vector

$$\begin{bmatrix} r_1 \\ \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \\ r_2 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{z}_2 \\ r_3 \\ \dot{x}_3 \\ \dot{y}_3 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -0.0141 \\ -0.0024 \\ -0.0048 \\ -0.0072 \\ -0.4490 \\ -0.0029 \\ -0.0045 \\ -0.0072 \\ -0.8930 \\ -0.0035 \\ -0.0041 \\ -0.0071 \end{bmatrix} \tag{10}$$

Multiplying the LOS unit vectors by their corresponding position magnitudes produces the following recreated position states.

$$[x_1^* \quad y_1^* \quad z_1^*]^T = [0.01194 \quad -0.007163 \quad -0.002388]^T$$
$$[x_2^* \quad y_2^* \quad z_2^*]^T = [-0.1214 \quad -0.2384 \quad -0.3603]^T \tag{11}$$
$$[x_3^* \quad y_3^* \quad z_3^*]^T = [-0.2822 \quad -0.4522 \quad -0.7170]^T$$

The initial position states differ from the final position states by the same proportional factor

$$\alpha_1 = \alpha_2 = \alpha_3 = -0.0024 \tag{12}$$

Next, this process was tested with noisy data. Specifically, the initial position states were perturbed with random Gaussian noise that had a zero mean and a standard deviation of 0.02. The noisy position states were

$$[x_1^+ \quad y_1^+ \quad z_1^+]^T = [-4.583 \quad 3.091 \quad 0.5915]^T$$
$$[x_2^+ \quad y_2^+ \quad z_2^+]^T = [49.25 \quad 98.78 \quad 153.93]^T \tag{13}$$
$$[x_3^+ \quad y_3^+ \quad z_3^+]^T = [116.4 \quad 190.1 \quad 299.0]^T$$

where the plus superscript designates that the data is corrupted.

Then the LOS unit vectors were found similarly to before. Because the data contains noise, the LOS unit vectors do not all fit the same trajectory exactly according to the HCW model. Therefore, plugging these LOS vectors into the POD method gives a 12X12 full rank matrix with no null vector. In this case, the approach used was to take the eigenvector associated with the smallest absolute eigenvalue to give the best relative orbit estimate. The eigenvalue was 0.0028 and the eigenvector was

$$\begin{bmatrix} r_1^+ \\ \dot{x}_1^+ \\ \dot{y}_1^+ \\ \dot{z}_1^+ \\ r_2^+ \\ \dot{x}_2^+ \\ \dot{y}_2^+ \\ \dot{z}_2^+ \\ r_3^+ \\ \dot{x}_3^+ \\ \dot{y}_3^+ \\ \dot{z}_3^+ \end{bmatrix} = \begin{bmatrix} -0.0196 \\ -0.0023 \\ -0.0048 \\ -0.0072 \\ -0.4505 \\ -0.0029 \\ -0.0032 \\ -0.0072 \\ -0.8924 \\ -0.0034 \\ -0.0041 \\ -0.0071 \end{bmatrix} \tag{14}$$

Multiplying the noisy LOS unit vectors by their corresponding position magnitudes produces the following recreated position states.

$$[x_1^* \quad y_1^* \quad z_1^*]^T = [0.0162 \quad -0.0102 \quad -0.0043]^T$$
$$[x_2^* \quad y_2^* \quad z_2^*]^T = [-0.1139 \quad -0.2411 \quad -0.3631]^T \tag{15}$$
$$[x_3^* \quad y_3^* \quad z_3^*]^T = [-0.2716 \quad -0.4550 \quad -0.7181]^T$$

The proportional factors between the noisy initial position states and the recreated position states are

$$[\alpha_1,\ \alpha_2,\ \alpha_3] = [-0.0035, \quad -0.0024, \quad -0.0024] \tag{16}$$

The first proportional factor differs from the proportional factor in Eq. (12) slightly, but the other two are exactly the same. This proves the POD method is useful for getting a good estimate of the correct deputy satellite relative orbit with noisy angle data.

BATCH FILTER

Then, the best estimate solution of the POD method is put into a batch filter that hones in on the correct initial state. This is accomplished by taking the result of the basis vector from the POD method, normalizing the states by the first value, the x position state, and then propagating it forward to a given set of times using the HCW model. At a given time, the propagated noisy position and velocity states are

$$\begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} = [\Phi(t_1, t_2)] \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ v_{x0} \\ v_{y0} \\ v_{z0} \end{bmatrix} \quad (17)$$

At each measurement time, the LOS unit vector represents two independent pieces of information (equivalent to two angles, e.g. azimuth and elevation). Thus, the LOS components in the y and z directions are

$$LOS_1 = \frac{y}{\sqrt{x^2+y^2+z^2}} \quad , \quad LOS_2 = \frac{z}{\sqrt{x^2+y^2+z^2}} \quad (18)$$

Therefore, the derivatives of these LOS components with respect to y and z are

$$\frac{d(LOS_1)}{dy} = \frac{1}{\sqrt{x^2+y^2+z^2}} - \frac{y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\frac{d(LOS_1)}{dz} = -\frac{yz}{(x^2+y^2+z^2)^{\frac{3}{2}}} \quad (19)$$

$$\frac{d(LOS_2)}{dy} = -\frac{yz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\frac{d(LOS_2)}{dz} = \frac{1}{\sqrt{x^2+y^2+z^2}} - \frac{z^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

The derivatives of these LOS components with respect to the velocity states are zero. Therefore, the measurement sensitivity matrix, which is a matrix of partial derivatives of the LOS components with respect to the states (often denoted by H), is shown below.

$$H = \begin{bmatrix} \frac{d(LOS_1)}{dy} & \frac{d(LOS_1)}{dz} & 0 & 0 & 0 \\ \frac{d(LOS_2)}{dy} & \frac{d(LOS_2)}{dz} & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

(Note that the H matrix is only 2X5 instead of 2X6, because only the last 5 states of the trajectory are being differentially corrected.) Then the H matrix is multiplied by the last 5X5 of the HCW state transition matrix.

$$H^* = H * [\Phi(2:6,2:6)] \tag{21}$$

This H^* is then concatenated to previous H^* matrices from earlier time iterations as

$$\bar{H}_* = \begin{bmatrix} H_{j-1}^* \\ H_j^* \end{bmatrix} \tag{22}$$

which will finally give a (2*j) X 5 matrix where j is the total number of time iterations. Also, LOS_1 and LOS_2 from equation 18 are placed in Y_{calc} as

$$Y_{calc} = \begin{bmatrix} LOS_1 \\ LOS_2 \end{bmatrix} \tag{23}$$

In addition to this, the true initial state is propagated forward in time as well. The measured LOS vector magnitudes in the y and z directions are placed in Y_{meas} as

$$Y_{meas} = \begin{bmatrix} LOS_{1,meas} \\ LOS_{2,meas} \end{bmatrix} \tag{24}$$

Then the calculated and measured LOS values are subtracted to give the residual

$$\Delta Y = Y_{meas} - Y_{calc} \tag{25}$$

The ΔY value at a given time is concatenated to previous ΔY values at earlier times as

$$\overline{\Delta Y} = \begin{bmatrix} \Delta Y_{j-1} \\ \Delta Y_j \end{bmatrix} \tag{26}$$

Once all of the iterations of time are finished, ΔX, which is a 5X1 vector used to adjust the last five values of the initial state guess towards the true initial state, is found as

$$\Delta X = \beta\ (\bar{H}_*^T * \bar{H}_*)^{-1} (\bar{H}_*^T * \overline{\Delta Y}) \tag{27}$$

where β is a scaling factor to make sure the ΔX step taken is adequate. β defaults to a value of one. If the previous adjustment to the initial state did not decrease the root-mean-square (RMS) of $\overline{\Delta Y}$ in the next iteration, β is lowered to take a smaller step. After the corrected initial state starts decreasing the RMS of $\overline{\Delta Y}$ again, β is reinitialized to one. When the batch filter converged on a solution with a minimum RMS of $\overline{\Delta Y}$, the filter was exited and the solution was taken as the best estimate for the basis vector.

The new corrected initial state is

$$\begin{bmatrix} 1 \\ y_j \\ z_j \\ v_{x_j} \\ v_{y_j} \\ v_{z_j} \end{bmatrix} = \begin{bmatrix} 1 \\ y_{j-1} \\ z_{j-1} \\ v_{x_{j-1}} \\ v_{y_{j-1}} \\ v_{z_{j-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta y \\ \Delta z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} \tag{28}$$

This procedure is repeated until the initial state guess converges to a final estimate of the true basis vector.

This batch filter was tested with several methods of determining the simulated measurements. First, the measurements were obtained using a true initial state that was propagated out using the HCW equations to a certain number of measurement times. The states at each measurement time were converted to a LOS vector, as shown in equation 18. Then Gaussian noise was added to these measured LOS vectors to simulate corrupted data. Afterwards, the measured LOS vectors were renormalized to ensure their magnitude remained unity.

The second method of determining the simulated measurements also used the HCW equations to obtain the measurements at the times needed. The difference in this method is the Gaussian noise was added to the azimuth and elevation angles, which were then converted to LOS measurements to use in the filter.

The third method of determining the simulated measurements used non-linear inertial two-body gravitational dynamics to get the states at the measurement times. The equations of motion (EOM) are shown below.

$$\ddot{\boldsymbol{r}} = -\frac{GM}{r^3}\vec{\boldsymbol{r}} \tag{29}$$

A circular orbit for the chief is required for the HCW model so a chief with zero eccentricity was specified in the initial condition. The differences between the inertial states of the chief’s orbit and the deputy’s orbit were then converted into the LVLH frame. The next step was to obtain LOS measurements for each LVLH relative motion state. After that, similarly to the first method, Gaussian noise was added to the LOS measurements to simulate corrupted data.

The fourth method was very similar to the third, with the only difference being the Gaussian noise was added to the azimuth and elevation angles instead of the LOS measurements. Table 1 summarizes the four methods of determining the measurements for the batch filter.

Table 1. Methods of Determining the Measurements for the Batch Filter.

	Equations used to obtain measurements	Gaussian noise added to
Method 1	HCW equations	LOS measurements
Method 2	HCW equations	Azimuth and Elevation angles
Method 3	Inertial two-body gravity equations	LOS measurements
Method 4	Inertial two-body gravity equations	Azimuth and Elevation angles

The following table shows the batch filter results when the POD method was used to determine the best estimate for the basis vector for the batch filter to correct on. The true initial state was [1, 2, 1, 0.1, 0.2, 0.1]. The Gaussian noise added to the simulated measurements had a 0.02 standard deviation and zero mean. The sample rate was 200 measurements over a quarter of the period of the chief's orbit. The residuals between the measurements and model will be shown for the first and last iteration of the batch filter for the highlighted case.

Table 2. Batch Filter results when the POD method was used for the initial guess.

MODEL USED TO GENERATE MEASUREMENTS, NOISE ADDED TO...	INITIAL GUESS FROM POD METHOD	# OF ITERATIONS	CONVERGED FILTER SOLUTION	RMS OF THE RESIDUALS
HCW, LOS	1.0000 1.8359 0.9751 -0.0006 -0.0014 0.0002	2	1.0000 1.8949 0.8858 0.0048 0.0070 0.0047	0.0233
HCW, Az & El	1.0000 1.8557 1.0476 0.0139 0.0253 0.0138	2	1.0000 1.8080 0.9392 0.0133 0.0241 0.0128	0.0161
2-Body, LOS	1.0000 2.1879 1.0901 -0.0010 -0.0044 -0.0010	25	1.0000 1.6428 0.6986 -0.0021 0.0002 0.0006	0.0330
2-Body, Az & El	1.0000 2.3678 1.2249 -0.0011 -0.0045 -0.0010	46	1.0000 1.9471 0.9153 -0.0022 0.0006 0.0007	0.0303

These results show that the POD method produces an initial guess for the batch filter that is significantly different from the true initial state. Therefore, the batch filter has a difficult task in converging to the true basis vector. It can be seen that the batch filter corrects towards the true basis vector, but the corrected initial state gets stuck in a local minima in the solution space so it does not converge to the true basis vector. In fact, the issue of the batch filter converging to local minima was a reoccurring problem unless the initial guess was only perturbed slightly in one state from the basis vector while the other states were kept at their true values, in addition to no noise being modeled in the measurements. This shows that the solution space is filled with these local minima. This makes it virtually impossible to converge to the true basis vector of the relative orbit, which is the global minima value.

Alternatively, the batch filter was also run without the POD method. In this case, the initial guess for the batch filter was provided that was slightly perturbed in the final five states. The results are shown in Table 3.

Table 3. Batch Filter results when the initial guess was arbitrarily provided.

MODEL USED TO GENERATE MEASUREMENTS, NOISE ADDED TO…	INITIAL GUESS	# OF ITERATIONS	CONVERGED FILTER SOLUTION	RMS OF THE RESIDUALS
HCW, LOS	1.0000 1.9000 1.1000 0.1200 0.2300 0.0800	4	1.0000 2.0678 1.0448 0.1125 0.2354 0.1194	0.0208
HCW, Az & El	1.0000 1.9000 1.1000 0.1200 0.2300 0.0800	2	1.0000 2.0424 1.0768 0.1136 0.2255 0.1128	0.0158
2-Body, LOS	1.0000 1.9000 1.1000 0.1200 0.2300 0.0800	5	1.0000 38.4001 20.1210 -0.3676 0.4273 0.2020	0.0576
2-Body, Az & El	1.0000 1.9000 1.1000 0.1200 0.2300 0.0800	5	1.0000 34.9080 17.8702 -0.3440 0.3909 0.1962	0.0546

It can be seen that the batch filter has better results in converging towards the true initial state when an arbitrary initial guess was provided and the HCW EOM were used to generate the measurements. It generally converged faster or in the same number of iterations as in the case where the POD method provided the initial guess for the basis vector. The RMS of the residuals is practically the same for both methods of determining the initial guess. This again shows that the batch filter is converging to a local minimum with approximately the same function value as when the POD method was used.

The residuals between the corrupted LOS measurements and the model were plotted in Figure 2 for the first iteration and the last iteration of the batch filter when the HCW equations were used to generate the measurements, the initial guess was arbitrarily provided, and noise was added to the azimuth and elevation angles.

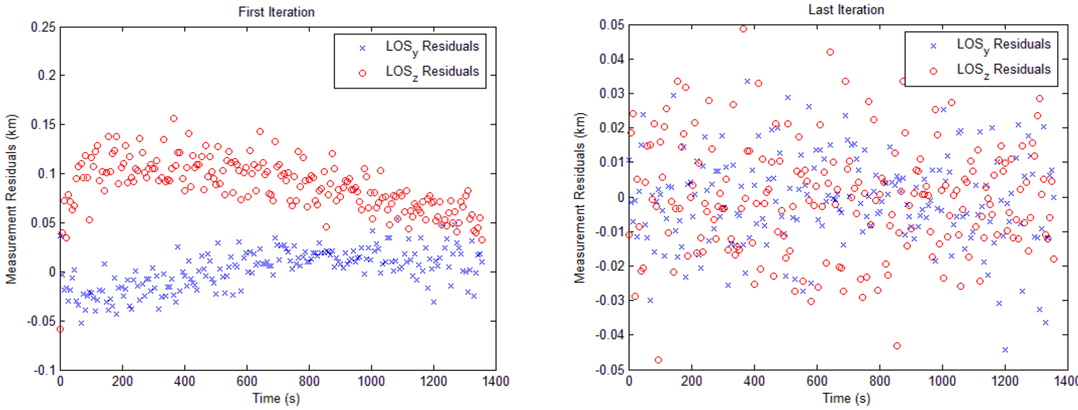

Figure 2. Residuals of the First and Last Iteration when using HCW to Generate the Measurements and Adding Noise to the Azimuth and Elevation Angles

In the first iteration of the batch filter, the residuals of the y and z components of the LOS unit vectors are separated into two groups. In the final iteration, these residuals are more averaged out and the values of the residuals are significantly reduced.

It was also found that varying the sample rate of the measurements did not affect the residual distribution. Sample rates with as low as 50 and as high as 400 measurements over a quarter of the chief's orbital period were examined.

In the case where inertial two-body gravitational dynamics was used to generate the measurements, the initial guess was given by the POD method, and noise was added to the azimuth and elevation angles, the following figure shows the residuals after the first and last iterations.

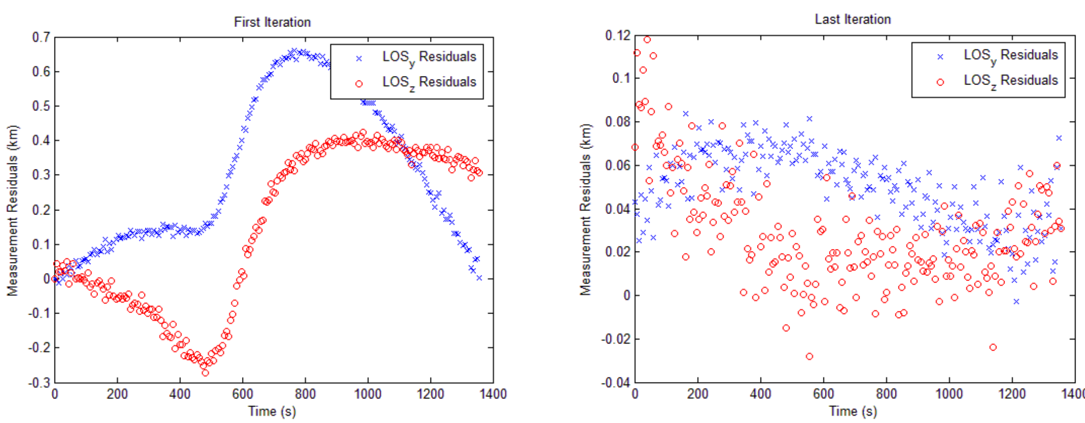

Figure 3. Residuals of the First and Last Iteration when using Inertial 2-Body Dynamics to Generate the Measurements and Adding Noise to the Azimuth and Elevation Angles

It is evident in the first iteration plot that there is a definite pattern in the residuals for each component. This shows that the initial guess for the basis vector given by the POD method is not ideal. Also, it can be seen in the last iteration plot of Figure 3 that the batch filter does not do as well in converging to a basis vector that completely smoothes out the residuals in the y and z components of the LOS unit vectors in this case. This is caused by using two different methods to propagate the basis vector. The measurements are generated with the more accurate nonlinear two body gravitational dynamics EOM, while the model is propagated using the linear HCW EOM. However, the batch filter does reduce the magnitude of the residuals in the last iteration compared to the first iteration by a factor of seven.

EXTENDED KALMAN FILTER

An Extended Kalman Filter[4] (EKF) was implemented next. The EKF was necessary because a regular Kalman filter assumes a linear model and linear measurements, while in this case, a linear model is used but the measurements are non-linear, as shown in Eq. (18).

For the EKF, it was assumed that no a priori information was known. Therefore, the initial covariance matrix, P_0, was a 5X5 identity matrix. It was assumed the model had no process noise, and the measurements had noise with a covariance of

$$R_0 = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \tag{30}$$

Then the initial state was propagated forward one time step. The predicted state covariance estimate was found as

$$\bar{P}_1 = \Phi(2{:}6, 2{:}6) * P_0 * \Phi(2{:}6, 2{:}6)^T \tag{31}$$

The LOS unit vectors and H matrix were found in the same manner as the batch filter methodology in Eqns. (18) and (20), respectively. Then, the residual, ΔY, between the measurements and the model derived states was found using Eq. (25). Accordingly, the Kalman gain was

$$K_1 = \bar{P}_1 H_1^T (H_1 \bar{P}_1 H_1^T + R_0)^{-1} \tag{31}$$

Finally, the updated state estimate and updated covariance estimate were

$$\begin{bmatrix} 1 \\ y_j \\ z_j \\ v_{x\,j} \\ v_{y\,j} \\ v_{z\,j} \end{bmatrix} = \begin{bmatrix} 1 \\ y_{j-1} \\ z_{j-1} \\ v_{x\,j-1} \\ v_{y\,j-1} \\ v_{z\,j-1} \end{bmatrix} + \begin{bmatrix} 0 \\ K_1 \Delta Y \end{bmatrix}_{6X1} \tag{32}$$

$$P_1 = [I - K_1 H_1] * \bar{P}_1 * [I - K_1 H_1]^T + K_1 R_0 K_1^T \tag{33}$$

Before the next iteration, the covariance estimate was reinitialized as

$$P_0 = P_1 \tag{34}$$

Then the state variables were propagated forward another time step and the process was continued until all of the measurements were processed. As with the batch filter, the EKF was tested with the four cases shown in Table 3. The sample rate was 200 measurements taken over a quarter of the chief's orbit period and Gaussian noise was added to the simulated measurements that had a 0.02 standard deviation and zero mean. The results using an arbitrary initial guess are shown in Table 4.

Table 4. EKF results when the initial guess was arbitrarily provided.

MODEL USED TO GENERATE MEASUREMENTS, NOISE ADDED TO...	INITIAL GUESS						CONVERGED FILTER SOLUTION					
HCW, LOS	1.0000	1.9000	1.1000	0.1200	0.2300	0.0800	1.0000	-4.0980	-0.3811	0.2342	0.1876	0.0872
HCW, Az & El	1.0000	1.9000	1.1000	0.1200	0.2300	0.0800	1.0000	-4.0128	-1.2460	-0.0772	0.2571	0.1111
2-Body, LOS	1.0000	1.9000	1.1000	0.1200	0.2300	0.0800	1.0000	-2.0470	-0.8304	0.1169	0.4006	0.1525
2-Body, Az & El	1.0000	1.9000	1.1000	0.1200	0.2300	0.0800	1.0000	-0.5395	-0.1209	0.0603	0.1307	0.0338

It can be seen in the table that the results from the EKF are not as expected currently. This is likely due to errors in the implementation of the EKF.

CONCLUSIONS

The goal of this work was to examine how accurately a basis vector for a family of relative orbit trajectories could be obtained from angles-only measurements. A POD method was introduced that took in three angle measurements that were given in the form of LOS vectors and produced an initial guess of the basis vector for the batch filter or EKF to correct on. The batch filter was run with different methods of generating the measurements, along with varying the initial guess.

Unfortunately, the solution space for the problem is filled with local minima. Therefore, the batch filter virtually never converged to the true basis vector unless the initial guess for the basis vector was contrived to only differ by a small amount in one state while the other states were kept at their true values, and no noise was added to the measurements. In any other case, the batch filter almost always converged to a local minimum between the initial state guess and the true basis vector.

FUTURE WORK

Going forward, the first task is to correct the errors in the EKF. After that, a comparison can be made between the batch filter and EKF to see which is a better method for solving this problem. Additionally, the use of the Tschauner-Hempel (TH) equations to model the dynamics, which do not require the chief's orbit to be circular, should be explored. Another avenue of research is using the relative orbit information to back out inertial orbit information to supplement the ephemeris data. From the HCW equations, the semi-major axis of the chief's inertial orbit can be found. From the TH equations, the semi-major axis and the eccentricity of the chief's inertial orbit can be found. Finally, if the chief and deputy satellites are able to communicate with each other, a request can be made for the deputy to perform a specified thrusting maneuver which will make it possible to determine which unique relative orbit the deputy is on with respect to the chief.

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