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REGRESSION ANALYSIS OF LAUNCH VEHICLE PAYLOAD CAPABILITY FOR INTERPLANETARY MISSIONS

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During the conceptual design of interplanetary space missions, it is common for engineers and mission planners to perform launch system trades. This paper provides an analytical means for facilitating these trades rapidly and efficiently using polynomial equations derived from payload planner's guides. These equations model expendable launch vehicles' maximum payload capability as a function of vis-viva energy (C3). This paper first presents the motivation and method for deriving these polynomial equations. Next, 34 polynomials are derived for vehicles among nine launch vehicle series: Atlas V, Delta IV, Falcon 9, and Taurus, as well as H-IIA, Long March, Proton, Soyuz, and Zenit. The quality of fit of these polynomials are assessed, and it is found that the maximum 95th percentile model fit error for all 34 vehicles analyzed is 4.43% with a mean of 1.44%, and the minimum coefficient of determination (R^2) is 0.99967. As a result, the equations are suitable for launch vehicle trade studies in conceptual design and beyond. A realistic example of such a trade for the Mars Reconnaissance Orbiter mission is provided.

I. INTRODUCTION

A common requirement in systems analysis and conceptual design for new spacecraft is the capability to perform rapid, parametric assessments of launch vehicle options. Such assessments allow engineers to incorporate launch vehicle considerations into cost, mass, and orbit performance trade studies early during conceptual design and development phases. Such launch vehicle analysis is traditionally accomplished through manual references to sources such as launch-vehicle-specific payload planner's guides. This method can be time consuming and is not conducive to parametric exploration and trade studies. In this paper, we derive least-square polynomials describing payload capability for a large set of expendable launch vehicles in order to enable more efficient launch option analyses for interplanetary mission applications.

Least squares regression provides a means of mathematically modeling a given data set by determining model parameters (e.g., polynomial coefficients). One advantage to this approach is that large sets of discrete data can be modeled using a simple equation. In this paper, we use polynomial equations that minimize the sum of the squares of residuals (i.e., a least-squares fit) to relate required vis-viva energy (C3) to maximum launch vehicle payload capability.

In this paper, least-square polynomials are derived for 34 different launch vehicles and their derivatives: Atlas V, Delta IV, Falcon 9, and Taurus, as well as H-IIA, Long March, Proton, Soyuz, and Zenit. The results here provided should be useful to spacecraft systems engineers and mission planners in allowing integrated, extensive, and efficient launch options analyses and parametric trade studies (of cost and payload mass, for example) early during conceptual design and development phases.

II. FITTING METHOD

When choosing a launch vehicle, mission planners often compare the maximum payload capabilities of a variety of different launch vehicles in order to choose the most appropriate vehicle for the mission. For interplanetary missions, the launch vehicle capability is a function of the vis-viva energy (C3) which is defined in Eq. [1] as the square of the hyperbolic excess velocity (v_∞) or twice the specific energy.

$$C3 \equiv v_{\infty}^{2} = 2E \quad [1]$$

For most commercially available launch vehicles, the maximum payload capability of a particular launch vehicle as a function of C3 energy is generally found in publicly available payload planner's guides in graphical or tabular form. The process of manually looking up maximum payload capability given a C3 value can be time consuming and inefficient when comparing many different launch vehicles. Fig. 1 shows a typical payload capability versus C3 curve found in a payload planner's guide (in this case, for the Atlas V 431 launch vehicle).

As shown in Fig. 1, if a mission planner for example needs to attain a C3 of 20 km²/s² to successfully deliver a payload, the Atlas V 431 vehicle could deliver a maximum of approximately 3,800 kg. If the required C3 value is 40 km²/s², the maximum payload the Atlas V 431 can carry is approximately 2,600 kg. If the mission planner wishes to compare the Atlas V 431 with other available launch vehicles, this look-up process would need to be repeated for each additional launch vehicle. This process can be simplified considerably through the use of a polynomial regression to represent the data in the plot.

This section includes an overview of the least squares regression process used to determine the payload capability polynomial equations for all launch vehicles discussed in this paper. Section II.I provides a summary of least squares polynomial fitting procedure while Section II.II describes the process used to determine the polynomial equations from the payload capability versus C3 curves found in payload planner's guides. The parameters used to assess the quality of fit of the equations are also discussed in Section II.II.

II.I Summary of Least Squares Polynomial Fitting

For a set of data points, a least-squares univariate regression establishes a relationship between a dependent and independent variable. This relationship can aid in determining the behavior of a complex set of data or an input variable whose output behavior is previously unknown. The general form of an *n*th order polynomial to which data may be fit is shown in Eq. [2]:

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_n x^n = \sum_{i=0}^{n} \alpha_i x^i \quad [2]$$

In Eq. [2], y represents the dependent (output) variable, α_0 through α_n represent the polynomial coefficients, and x through x_n represent the regression variables. For this paper we use a fourth-order polynomial of the form shown in Eq. [3]. A more detailed description of the rationale for choosing the fourth-order polynomial as opposed to other polynomial orders is provided in Appendix B of this paper.

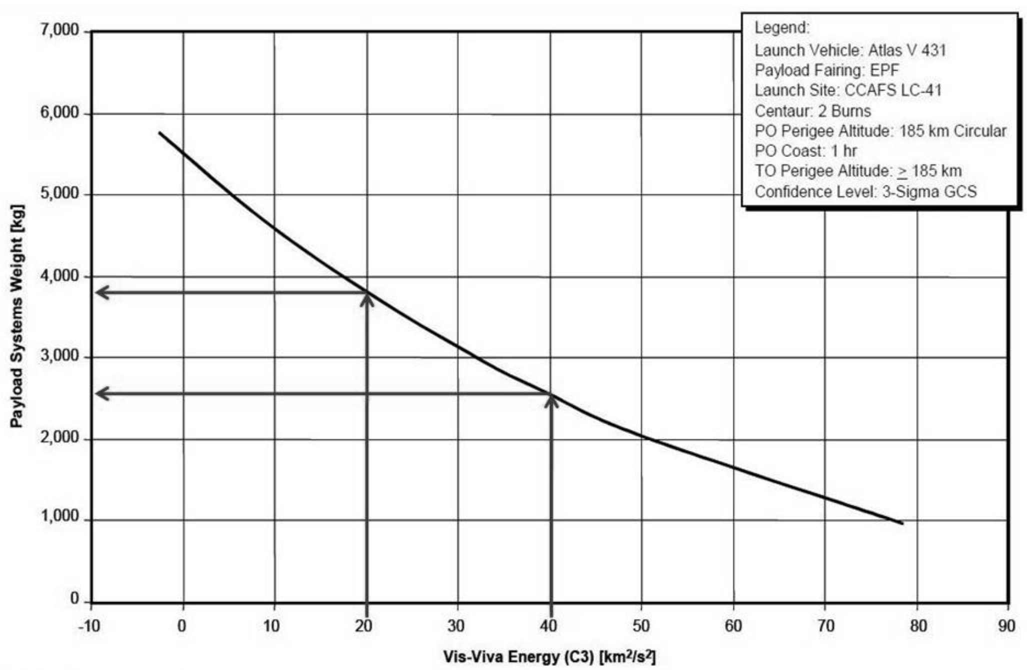

Fig. 1: Payload capability for Atlas V 431 as a function of C3 adapted from Ref. 1.

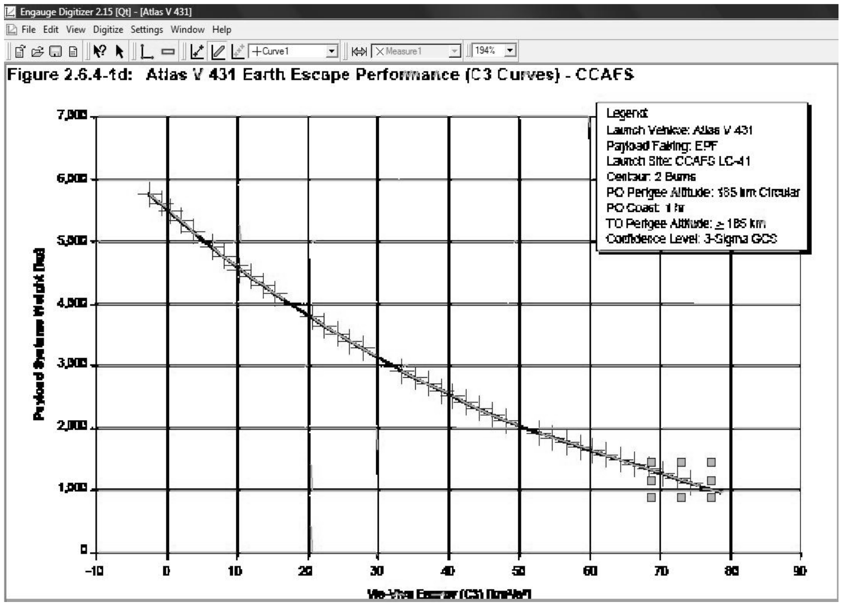

Fig. 2: Engauge Digitizer user interface for the Atlas V 431 figure-to-table conversion. *Engauge automatically detects line segments in the image and allows the user to export the data into a table.*

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4 \quad [3]$$

Since the output variable or interest is the maximum payload mass that can be carried, we denote this variable as m_{pay} and the input variable is the vis-viva energy which is denoted as *C3*. Eq. [4] shows the fourth order polynomial incorporating this notation:

$$m_{pay} = \alpha_0 + \alpha_1 C3 + \alpha_2 C3^2 + \alpha_3 C3^3 + \alpha_4 C3^4 \quad [4]$$

The following subsection describes the determination of the polynomial coefficients and the analysis of the quality of fit of the least square polynomial equations using the Atlas V 431 launch vehicle as an example case. Section III provides the polynomial equations and a summary of the statistical metrics used to characterize the quality of fit for the remaining 33 launch vehicles.

II.II Least Squares Polynomial Fitting Procedure

To determine the polynomial coefficients for each launch vehicle, a standardized fitting process is used. The process consists of three main steps: (1) digitize the data contained in the plots from payload planner's guides, (2) use MATLAB to derive a least-squares polynomial for the data obtained in the previous step, and (3) check the quality of fit of the polynomial using statistical tests.

In the first step, Engauge Digitizer software is used to convert images of plots into a numerical table of data points. For the Atlas V, Delta IV, Falcon 9 and Taurus launch vehicles, these plots were obtained from publicly available payload planner's guides specific to each launch vehicle.[2,3,4,5] For the H-IIA, Long March, Soyuz, Proton and Zenit vehicles, the plots were obtained from Ref. 6. Engauge automatically recognizes line segments within an imported image and allows the user to select which line segments to convert to an exportable data table. Fig. 2 shows the Engauge user interface in the process of highlighting line segments and selecting them to export data points.

Second, the table created by the Engauge Digitizer software is imported into the MathWorks MATLAB software package. MATLAB is used to perform a least squares regression to determine the polynomial coefficients (α_i values) corresponding to the fourth-order model shown in Eq. [4]. This process can be performed for any launch vehicle, given a table of its C3 values and corresponding payload capability values. The quality of the fit produced using this method can then be assessed by examining several statistical metrics including the coefficient of determination, R^2. For this paper, a total of three parameters are used to examine the quality of fit: R^2 value, the comparison of actual payload versus C3 and predicted payload versus C3 plots, and model fit error (MFE).

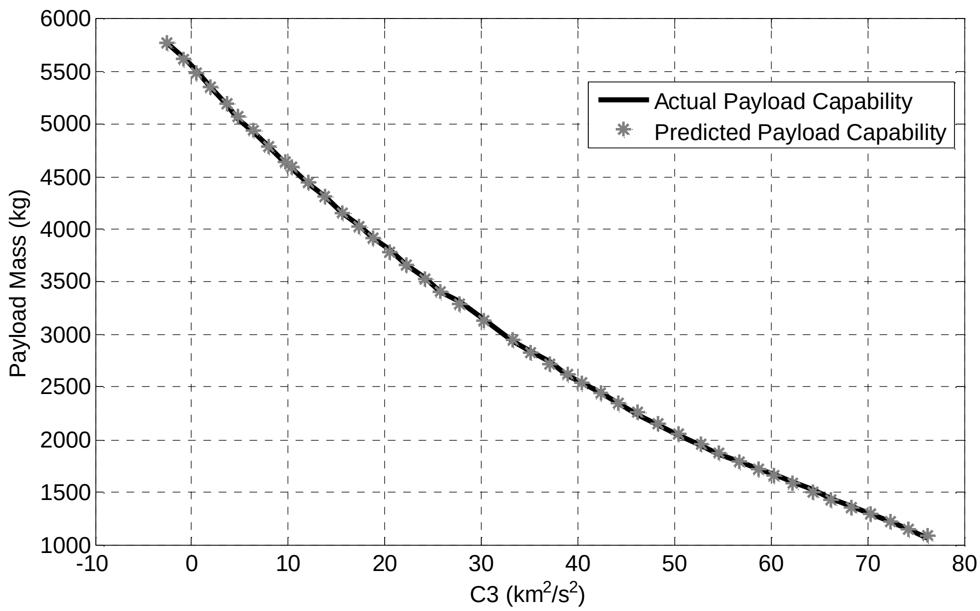

Fig. 3: Actual and predicted payload versus C3 plots for the Atlas V 431 vehicle.

The first quality of fit test, the coefficient of determination (R^2) of the fit, is a measure of the proportion of the sum of the squares of the errors to the total sum of squares. It indicates how well the assumed functional form of the model accounts for the variability of the data. An R^2 value close to unity indicates that the model properly captures the variability of the data set. For the model presented in this paper, the R^2 value is appropriate for because there are many more data points than degrees of freedom in the model (five, as indicated by the number of α values in Eq. [3]). The R^2 values for all fits in this paper are greater than 0.99967 and the R^2 value for the Atlas V 431 fit is 0.99993, indicating that the chosen model well describes the overall variability of payload capability with C3.

The second test for quality of fit is a visual comparison of actual payload versus C3 and predicted payload versus C3 plots. When plotted on the same axes, these data should exactly coincide if the model perfectly predicts the output. Figure 3 shows this data for the Atlas V 431 vehicle. The solid black line shows the actual payload capability versus C3 curve as predicted by the fourth order model and the gray asterisks show the predicted payload capability versus C3 curve. As shown in Fig. 3, slight variations between the two curves are discernible but overall they overlap significantly, visually indicating a good quality of fit for the Atlas V 431 example case.

The third quality of fit test is detailed quantitative examination of model fit error (MFE). The MFE test consists of analyzing (1) the distribution of the residuals, (2) the normal plot of the residuals, and (3) the residuals represented as percentages of actual payload capability (percent error). The residual is the difference between the actual payload capability and the payload capability predicted by the model. Ideally, the residuals will be small and randomly distributed, independent of the predicted output. Fig. 4 shows the residual versus payload capability plot for the Atlas V 431 vehicle. The residuals are small, all less than 25 kg in magnitude (compare with the actual payload of 1000–6000 kg; more on this point in Fig. 6-7 and the corresponding text) and are randomly distributed with no clear pattern.

Another indication of quality of fit is the normal plot of the residuals. A normal plot shows the probability of occurrence of a data point versus the numerical value of the data point. A reference line representing the normal distribution is plotted on the same axes. For a perfectly normal distribution, the data points will lie directly along the reference line, indicating the data is normally distributed. Fig. 5 shows the normal plot for the residuals for the Atlas V 431 vehicle. The dashed line corresponds to a normal distribution. The crosses lie in close proximity to the dashed line with the exception of several points near each end of the line, specifically in the 15 kg to 25 kg and 15 kg to 20 kg ranges. Overall, the close proximity of the crosses to the dashed reference line indicates the residuals are nearly normally distributed.

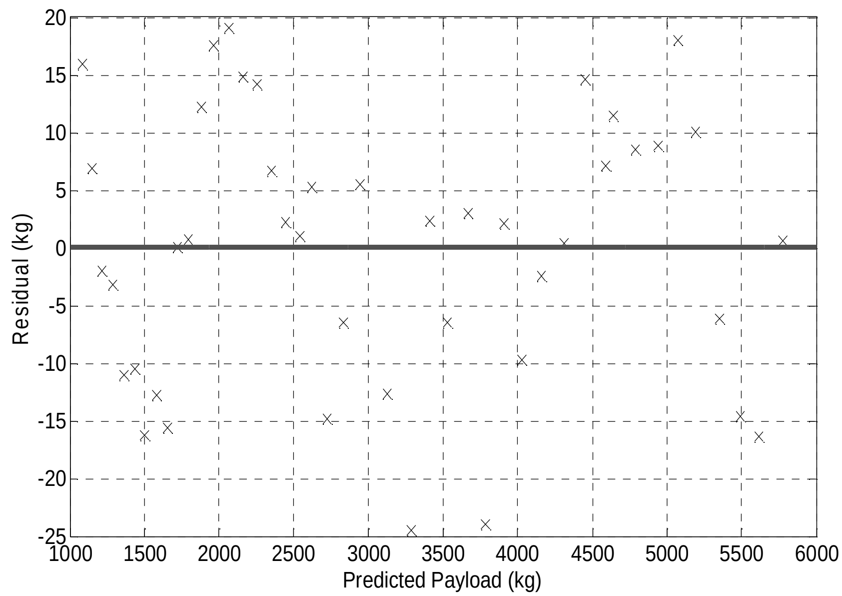

Fig. 4: Residual distribution for Atlas V 431 vehicle.

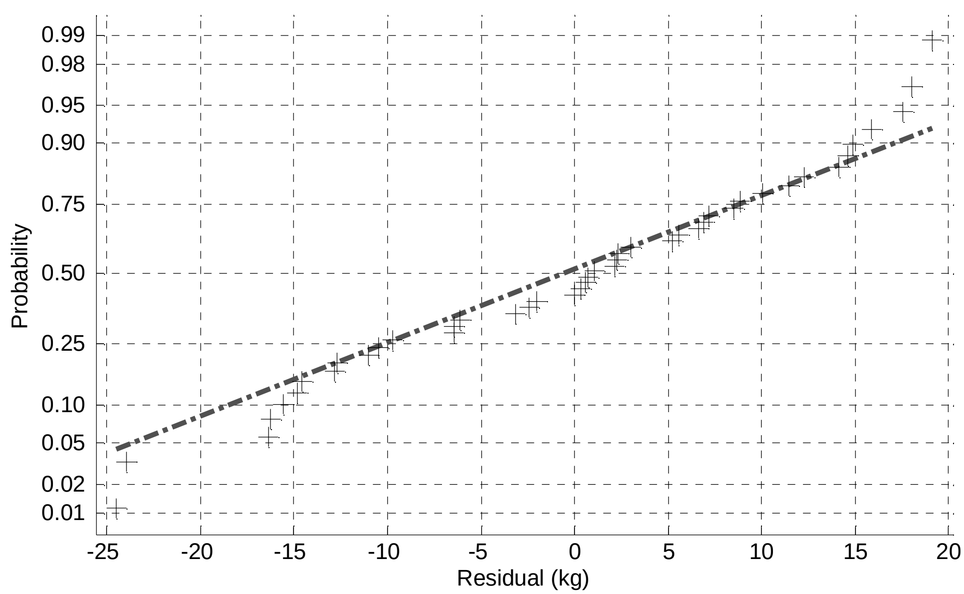

Fig. 5: Normal plot for the Atlas V 431 vehicle.

The third component of the MFE test for quality of fit involves examining the percent error distribution. For this metric the residuals are represented as percentages of the actual payload capability (percent errors) and are plotted as a histogram. Fig. 6 shows the percent error distribution for the Atlas V 431 vehicle. The percent errors are evenly distributed about the zero percent line, indicating a good fit. Also note that neither mean nor median deviate from zero by more than 0.04 percent. Figure 7 shows the distribution of the absolute value of the MFE with 90th, 95th and 100th (maximum) percentiles marked. This indicates that for the Atlas V 431 the model correctly predicts the payload capability to within an accuracy of 0.9 percent for 90 percent of the data points. The maximum percent error for the model is 1.48 percent, signifying a highly accurate prediction model when used for applications that can withstand a tolerance of approximately 0.9 to 1.5 percent error. These percentiles are included in the next section for all of the launch vehicles examined in this paper.

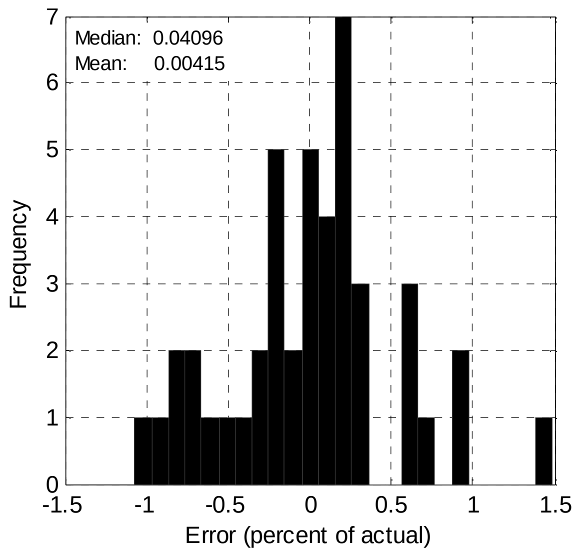

Fig. 6: Percent error distribution for the Atlas V 431 vehicle.

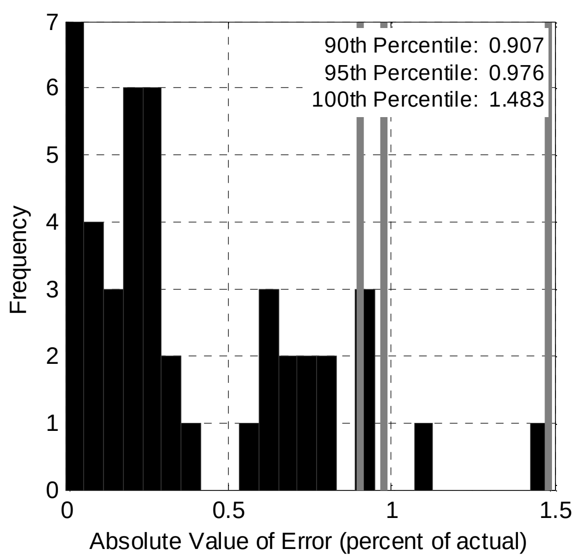

Fig. 7: Absolute value of percent error distribution for Atlas V 431. *The gray lines indicate the 90th, 95th and 100th percentile errors.*

III. RESULTING POLYNOMIAL EQUATIONS

The fitting procedure described above was repeated for 34 different launch vehicles including the Atlas V, Delta IV, Falcon 9 and Taurus as well as the H-IIA, Long March, Proton, Soyuz, and Zenit series. Polynomial equations were developed and analyzed for all 34 launch vehicles. The payload capability equation for the Atlas V 431, which was derived and analyzed in the previous section, is given in Eq. [5] where *C3* is given in km²/s² and m_{pay} in kg:

$$\begin{aligned} m_{pay} &= 5.531\times10^{3} - 96.41\cdot C3 \\ &+ 0.5433\cdot C3^{2} + 1.324\times10^{-3}\cdot C3^{3} \\ &- 2.484\times10^{-5}\cdot C3^{4} \end{aligned} \quad [5]$$

Table 1 shows the payload capability equation coefficients for all launch vehicles analyzed and the C3 ranges for which the polynomial equations are valid. Table 2 shows the results of the quality of fit analysis for all of the launch vehicles analyzed in this paper. This table includes a summary of the error statistics including, R^2 value and 95th percentile model fit error as a percentage of the total model fit error and in kilograms.

As Tables 1 and 2 show, all R^2 values are greater than 0.99967. The minimum number of points used in the fitting procedure was 16, which is more than three times larger than the minimum number of points necessary (five) for the fit. The mean number of data points used in the fitting procedure is twelve times higher than the minimum, at approximately 60 points. Also shown in Table 2, the maximum 95th percentile MFE as a percentage of the total is 4.43% and the mean 95th percentile MFE is substantially smaller at 1.44%. Additionally, the maximum 95th percentile MFE in kilograms is 46.3 kg with the average being much smaller at 19.3 kg. These high R^2 and low MFE values illustrate that the polynomial regressions are indeed accurate and appropriate for conceptual design and trade studies.

The C3 ranges used in the fitting procedure depend on the information given in the source data. For the launch vehicles analyzed in this paper, the C3 values ranged on average from -13 km²/s² to 76 km²/s². The Atlas and Delta payload planner's guides provided a wider range from approximately -20 to 120 km²/s², and the Long March and Soyuz vehicle plots did not provide any negative (i.e., lower-than-escape-energy) C3 values.

Vehicle	Least Squares Polynomial Coefficients, cf. Eq. [4]					C3 Range (km²/s²)	
	α_0	α_1	α_2	α_3	α_4	Min	Max
Atlas V 401	3.357×10^{3}	-63.46	0.4948	-3.2×10^{-3}	1.22×10^{-5}	-12.74	77.68
Atlas V 411	4.280×10^{3}	-79.75	0.6819	-4.2×10^{-3}	1.19×10^{-5}	-10.08	90.38
Atlas V 421	4.855×10^{3}	-87.43	0.6757	-3.9×10^{-3}	1.37×10^{-5}	-10.96	90.36
Atlas V 431	5.531×10^{3}	-96.41	0.5433	1.3×10^{-3}	-2.48×10^{-5}	-2.54	76.24
Atlas V 501	3.056×10^{3}	-56.95	0.2466	9.3×10^{-4}	-1.04×10^{-5}	-19.64	73.44
Atlas V 501 with STAR 48V OIS	3.107×10^{3}	-56.63	0.6353	-4.8×10^{-3}	1.56×10^{-5}	-29.21	121.06
Atlas V 511	4.017×10^{3}	-71.37	0.4438	-1.2×10^{-3}	-9.15×10^{-7}	-19.34	94.38
Atlas V 511 with STAR 48V OIS	4.053×10^{3}	-73.51	0.7925	-5.6×10^{-3}	1.82×10^{-5}	-29.71	118.83
Atlas V 521	4.812×10^{3}	-82.40	0.5263	-1.5×10^{-3}	-3.73×10^{-7}	-20.16	99.84
Atlas V 521 with STAR 48 V OIS	4.912×10^{3}	-87.04	0.8322	-4.6×10^{-3}	1.09×10^{-5}	-29.06	145.78
Atlas V 531	5.586×10^{3}	-100.14	0.8596	-5.0×10^{-3}	1.39×10^{-5}	-30.23	107.23
Atlas V 531 with STAR 48V OIS	5.512×10^{3}	-91.63	0.7767	-3.7×10^{-3}	7.78×10^{-6}	-20.04	157.85
Atlas V 541	6.203×10^{3}	-110.39	1.0349	-7.3×10^{-3}	2.49×10^{-5}	-29.64	106.43
Atlas V 541 with STAR 48V OIS	6.147×10^{3}	-103.75	0.9108	-4.6×10^{-3}	9.89×10^{-6}	-19.76	163.48
Atlas V 551	6.541×10^{3}	-117.91	1.1419	-8.5×10^{-3}	3.07×10^{-5}	-29.59	103.41
Atlas V 551 with STAR 48V OIS	6.536×10^{3}	-112.72	1.0171	-5.2×10^{-3}	1.14×10^{-5}	-20.05	154.00
Delta IV Heavy	1.040×10^{4}	-170.07	1.1109	-2.7×10^{-3}	-5.00×10^{-6}	-9.38	98.84
Delta IV M+(4,2)	4.646×10^{3}	-87.84	0.6525	-2.9×10^{-3}	5.03×10^{-6}	-9.57	84.67
Delta IV M+(5,2)	3.868×10^{3}	-76.76	0.4783	-9.0×10^{-4}	-1.00×10^{-5}	-9.57	64.74
Delta IV M+(5,4)	5.401×10^{3}	-93.25	0.5471	-2.1×10^{-3}	4.82×10^{-6}	-9.57	79.16
Delta IV Medium	3.302×10^{3}	-67.70	0.4784	-3.2×10^{-3}	1.95×10^{-5}	-9.69	65.39
Falcon 9	2.495×10^{3}	-71.41	0.6103	-4.1×10^{-3}	2.09×10^{-5}	-16.00	45.00
Taurus 2130	2.958×10^{2}	-7.61	0.1044	-2.5×10^{-3}	4.25×10^{-5}	-1.85	29.93
Taurus 2230	2.485×10^{2}	-6.71	0.0515	1.0×10^{-3}	-2.64×10^{-5}	-2.07	29.32
H-IIA H2A202-4S	2.634×10^{3}	-71.84	0.6161	-2.3×10^{-3}	-4.28×10^{-5}	-19.84	39.83
H-IIA H2A2022-5S	2.919×10^{3}	-75.92	0.6673	-9.4×10^{-4}	-8.16×10^{-5}	-19.85	40.01
H-IIA H2A2024-5S	3.285×10^{3}	-82.17	0.7400	-1.9×10^{-3}	-7.72×10^{-5}	-19.86	40.19
Long March LM-2E	2.383×10^{3}	-48.69	0.4500	-2.6×10^{-3}	5.25×10^{-6}	0.00	49.91
Long March LM-3A	1.450×10^{3}	-47.06	-0.4361	3.7×10^{-2}	-6.16×10^{-4}	0.55	29.67
Long March LM-3B	3.413×10^{3}	-81.02	-0.0512	2.6×10^{-2}	-3.49×10^{-4}	1.02	47.32
Long March LM-3C	2.329×10^{3}	-71.89	0.5289	2.1×10^{-3}	-9.03×10^{-5}	0.55	38.17
Proton M/Breeze M	5.654×10^{3}	-116.70	1.0049	-5.9×10^{-3}	2.53×10^{-5}	-5.00	65.00
Zenit 3-SL (0° declination)	4.007×10^{3}	-109.04	1.0031	-2.9×10^{-3}	-9.37×10^{-5}	-14.88	29.63
Zenit 3-SL (30° declination)	3.819×10^{3}	-105.42	0.9393	-3.3×10^{-3}	-4.22×10^{-5}	-17.93	29.51
Soyuz (Fregat delivered to orbit)	3.480×10^{2}	80.45	-4.2776	7.9×10^{-2}	-5.26×10^{-4}	25.98	43.74
Soyuz (suborbital separation of Fregat)	1.592×10^{3}	-36.14	-0.1896	1.6×10^{-2}	-1.01×10^{-4}	0.10	25.77

Table 1: Polynomial Equations and Ranges of Validity

Vehicle	R^2	Number of data points used in fit	95th percentile Model Fit Error (percent of actual)	95th percentile Model Fit Error (kg)
Atlas V 401	0.99999	78	1.0978	10.582
Atlas V 411	0.99999	81	1.1566	16.898
Atlas V 421	0.99989	60	2.2247	25.734
Atlas V 431	0.99993	44	0.9757	20.568
Atlas V 501	0.99994	31	2.4123	18.769
Atlas V 501 with STAR 48V OIS	0.99981	58	2.5766	38.803
Atlas V 511	0.99994	36	3.2286	23.963
Atlas V 511 with STAR 48V OIS	0.99987	48	4.4310	46.345
Atlas V 521	0.99996	38	1.9252	21.091
Atlas V 521 with STAR 48 V OIS	0.99994	52	2.5507	35.197
Atlas V 531	0.99997	57	1.3445	28.824
Atlas V 531 with STAR 48V OIS	0.99995	60	2.3346	25.916
Atlas V 541	0.99996	62	2.4136	32.531
Atlas V 541 with STAR 48V OIS	0.99994	73	2.2707	33.437
Atlas V 551	0.99995	65	1.9958	39.483
Atlas V 551 with STAR 48V OIS	0.99995	76	1.8626	31.081
Delta IV Heavy	0.99998	109	0.9903	26.981
Delta IV M+(4,2)	0.99998	87	0.7682	12.790
Delta IV M+(5,2)	0.99997	70	1.0239	13.558
Delta IV M+(5,4)	0.99998	86	0.5539	12.465
Delta IV Medium	0.99996	70	1.1902	10.278
Falcon 9	0.99967	17	1.8576	37.253
Taurus 2130	0.99995	98	0.4167	0.716
Taurus 2230	0.99994	97	0.4422	0.665
H-IIA H2A202-4S	0.99994	77	0.9468	16.584
H-IIA H2A2022-5S	0.99996	80	0.7993	12.331
H-IIA H2A2024-5S	0.99995	80	0.8768	15.651
Long March LM-2E	0.99991	62	0.6264	8.712
Long March LM-3A	0.99992	39	1.2903	6.037
Long March LM-3B	0.99986	67	1.7732	18.046
Long March LM-3C	0.99996	51	0.9908	7.213
Proton M/Breeze M	0.99997	16	0.7941	15.475
Zenit 3-SL (0° declination)	0.99998	48	0.3568	11.500
Zenit 3-SL (30° delcination)	0.99997	50	0.4749	12.665
Soyuz (Fregat delivered to orbit)	0.99978	20	0.6659	3.685
Soyuz (suborbital separation of Fregat)	0.99994	34	0.3183	3.263

Table 2: Payload Capability Determination Equation Quality of Fit Statistics

IV. EXAMPLE APPLICATION

This section presents an example application of the polynomial equations described in the previous section. In this example, a mission planner has information regarding payload mass and mission C3 requirements and uses the equations to determine the payload capability of several different launch vehicles. This allows the mission planner to make a comparison and choose the appropriate vehicle that will accommodate the payload plus potential future mass growth. A discussion of how the payload capability polynomial equations can be useful when considering cost and launch margin is also included in this section.

It is a common task during the conceptual design phase for mission planners to choose a launch vehicle given payload and mission requirements. As an example of how the polynomial equations developed in this paper can be useful in this situation, we examine launch vehicle selection for the Mars Reconnaissance Orbiter (MRO) mission. Launched on August 12, 2005, from Cape Canaveral, MRO's main goals were to investigate the Martian terrain and climate plus the influence of water on the environmental processes that have shaped Martian surface.[7] Mars Reconnaissance Orbiter had a launch mass of 2,180 kg, a launch vis-viva energy (C3) of 16.35 km²/s², and was launched on a Atlas V 401 launch vehicle.[7,8]

In the conceptual design for MRO, to determine candidate launch vehicle performance based on C3 energy, a mission planner would look up the maximum payload capability from the payload planner's guide in a similar fashion as in Fig. 8. The process would have to be repeated for all launch vehicles under consideration and would take a significant amount of time. When using the polynomial equations developed here (Table 1), a mission planner could use a program such as Microsoft Excel or MATLAB to quickly determine multiple launch vehicles' capabilities for a given set of C3 values. For the MRO mission, a mission planner could determine the capability of a variety of launch vehicles for a C3 of 16.35 km²/s² and compare the difference between the required payload and the launch vehicle's available payload capability. Any launch vehicle with a negative launch margin can be eliminated from consideration. Table 3 shows the results of the polynomial equations for the Atlas V 431, Atlas V 401, Soyuz (suborbital separation of Fregat), Delta IV Medium, Delta IV M+(5,2), and Proton M/Breeze M vehicles. Column 2 of Table 3 shows the capability of each vehicle, and columns 3 and 4 show the launch margin and launch margin as a percentage of required payload capability. Launch margin is defined as the difference between required and available payload capability. Note that the Soyuz vehicle has a negative margin, meaning it cannot carry the required payload mass. By contrast, the Atlas V 431 and Proton M/Breeze M vehicles can carry the desired payload with large margins of 83% and 88%, respectively. The Delta IV Medium can carry the required payload with a slight margin of 6%, and the Delta IV+M(5,2) can carry the required payload with a margin of approximately 25%. The Atlas V 401, which was the launch vehicle selected for this mission, can

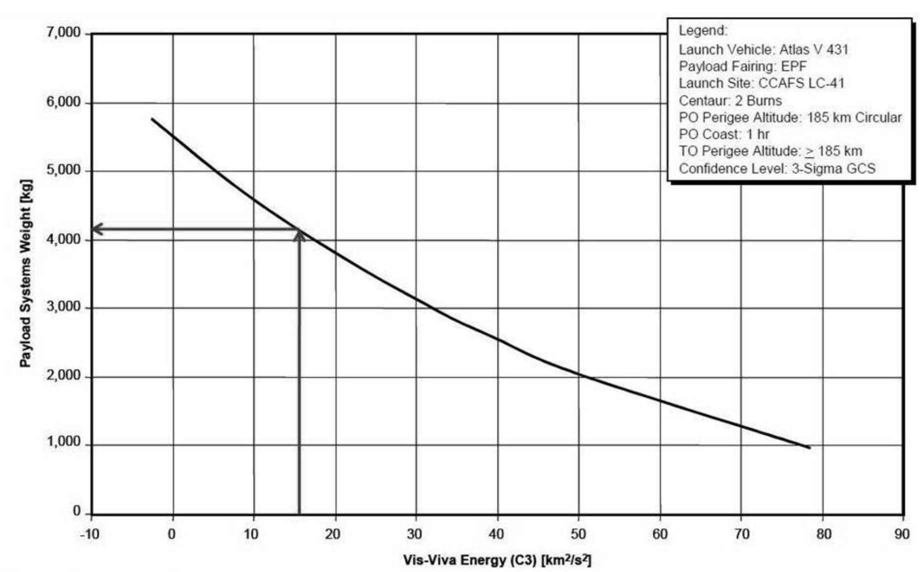

Fig. 8: Graphical payload capability look-up technique for MRO mission using the Atlas V 431 vehicle (adapted from Ref. 1).

Vehicle	Payload Capability (kg)	Margin (kg)	Margin (%)
Atlas V 431	4104	1924	88.26
Proton M/Breeze M	3990	1810	83.03
Atlas V 401	2438	258	11.83
Delta IV Medium	2311	131	5.99
Delta IV M+(5,2)	2736	556	25.50
Soyuz (suborbital sep. of Fregat)	1013	-1167	-53.53

Table 3: Payload Capability Polynomial-Equation-Determined Payload Capability and Margin for MRO Mission (C3 = 16.35 km^2/s^2)

carry the required payload with a margin of approximately 12%.

In addition to launch margin, another important consideration when choosing a launch vehicle is the launch cost. One tool that can be used to aid in the decision-making process is a Pareto front, which indicates the set of non-dominated points (for which one objective cannot be improved without sacrificing another). If cost and launch margin are to be considered, a Pareto front such as in Fig. 9 can be drawn to examine the trade between these two parameters. Fig. 9 shows the Pareto front for the launch vehicles considered in Table 3. Each vehicle is plotted as an “x” and the solid black line connecting the points represents the Pareto front. Fig. 9 shows that for a mission C3 of 16.35 km²/s², the Proton and Delta IV vehicles are dominated by the Atlas V vehicles. For example, the Atlas V 431 can provide a larger margin at a smaller cost than the Proton M/Breeze M, and the Atlas V 401 can provide a larger margin at the same cost as the Delta IV Medium. The polynomial equations developed in this paper (shown in Table 1) facilitate launch vehicle comparisons such as this.

V. CONCLUSION

This paper demonstrated a methodology and statistical results for fitting fourth-order polynomials to maximum payload capability versus C3 curves for a wide variety of launch vehicles, including the Atlas V, Delta IV, Falcon 9, and Taurus vehicles as well as the H-IIA, Long March, Proton, Zenit, and Soyuz vehicles. Of all vehicles considered, the maximum 95th percentile model fit error was 4.43%, with the mean at 1.44%. The minimum coefficient of determination (R^2) was 0.99967 considering all 34 launch vehicles analyzed, and the mean was slightly higher at 0.9999. These statistics demonstrate that the equations are appropriate for trade studies during the conceptual design phase and beyond. Because of their analytical nature, the polynomial equations allow mission planners to perform quick, efficient trade studies of which an example is discussed in this paper: selecting a launch vehicle given mission required C3 and payload. Future work includes expanding the equation database

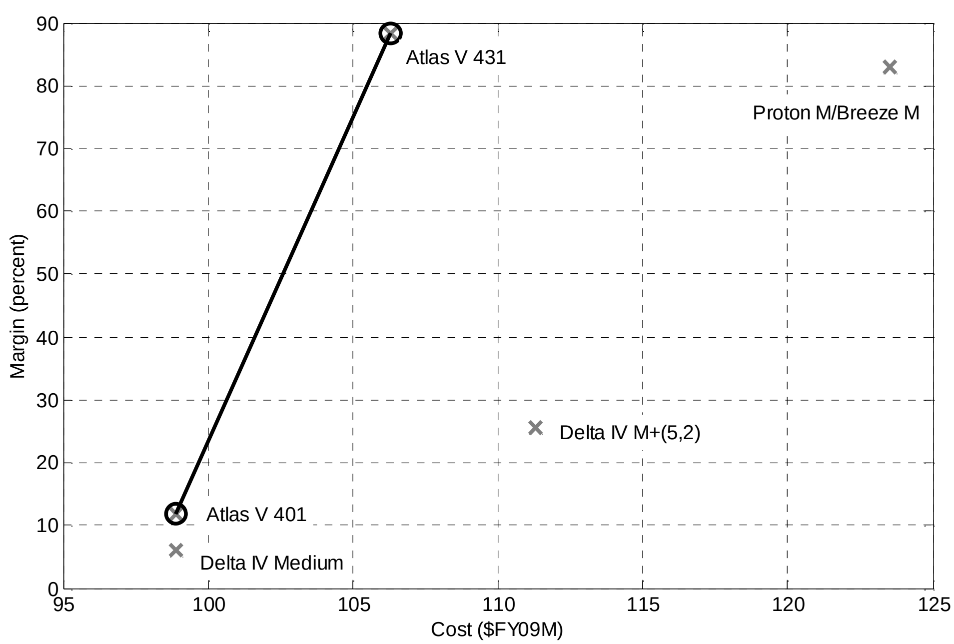

Fig. 9: Pareto front comparing launch vehicle cost and margin for a mission C3 of 16.35 km²/s².

to include additional launch vehicles, particularly the Ariane 5. Additionally, the creation of a user interface for easier navigation of the payload capability determination equation database is an important area for future work.

Overall, this work provides a powerful tool to the aerospace engineer during conceptual design and beyond. It is intended that the capabilities enabled by this work will aid the mission planner or project manager in making efficient, informed trades and decisions on launch options early during design and development for a wide variety of mission applications.

VI. REFERENCES

1. Atlas Launch System Mission Planner's Guide [online], Lockheed Martin Commercial Launch Services, Rev. 10a, Jan. 2007, Available: http://www.ulalaunch.com/docs/product_sheet/Atlas_Mission_Planner_14161.pdf [8 Oct. 2009].

2. Delta IV Payload Planners Guide [online], United Launch Alliance, LLC, Sept. 2007, Available: http://www.ulalaunch.com/docs/product_sheet/DeltaIVPayloadPlannersGuide2007.pdf [8 Oct. 2009].

3. Taurus Launch System Payload User's Guide [online], Orbital Sciences Corporation, Release 4.0, March 2006, Available: http://www.orbital.com/NewsInfo/Publications/taurus-user-guide.pdf [11 Nov. 2009].

4. Falcon 9 Launch Vehicle Payload User's Guide [online], Space Exploration Technologies Corporation, Rev. 1, 2009, Available: http://www.spacex.com/Falcon9UsersGuide_2009.pdf [11 Nov. 2009].

5. Proton Launch System Mission Planner's Guide [online], International Launch Services, Inc., Rev. 6, Dec. 2004, Available: http://www.ilslaunch.com/assets/pdf/pmpg_2e.pdf [8 Oct. 2009].

6. S.J. Isakowitz, J.B. Hopkins, and J.P. Hopkins, Jr., International Reference Guide to Space Launch Systems, 4th ed., American Institute of Aeronautics and Astronautics, Reston, 2004.

7. R.E. Lock, et al, The Mars Reconnaissance Orbiter Mission Plan, Advances in the Astronautical Sciences 119 (2005) 2629-2647.

8. J.E. Graf, R.W. Zurek, J.E. Howard, J. Benhan and M.D. Johnston, The Mars Reconnaissance Orbiter Mission, 56th International Astronautical Congress, 2005. IAC-05-A.3.3

APPENDIX A: NOMENCLATURE

C3	vis-viva energy
E	specific energy
MFE	Model Fit Error
m_{pay}	launch vehicle payload mass capacity
MRO	Mars Reconnaissance Orbiter
v_∞	hyperbolic excess velocity
x	regression variable
y	dependent (output) variable
α	polynomial coefficient

APPENDIX B: RATIONALE FOR CHOOSING A FOURTH-ORDER POLYNOMIAL

As mentioned in Section II.I of this paper, a fourth order polynomial was chosen for fitting for all launch vehicle data. It was determined that a fourth order polynomial would give the most appropriate fit by comparing the normal distribution and error ranges for different orders of polynomial. For selected launch vehicles, least square polynomials of order one through seven were used to determine the relationship between C3 and maximum payload capability. The absolute residual (in kilograms) for each order of polynomial was then plotted using a box plot like the one shown in Fig. 10 for the Atlas V 401 vehicle. The box for the first order polynomial is not included due to the extremely large range of absolute residuals which immediately eliminated this order of polynomial from consideration.

The box plot graphically depicts the maximum and minimum absolute residual (indicated by the vertical whiskers extending from the top and bottom of each box), the 25th and 75th percentiles (represented by the bottom and top boundaries of the box, respectively), and the 50th percentile (indicated by the horizontal line contained within the boundaries of each box). A smaller range of absolute residuals indicates a better quality of fit. Additionally, the spacing of the segments on each box can help determine if the residuals are symmetrically distributed. For a perfectly symmetrically distributed set of residuals, the spacing between the top whisker and top boundary of the box and the bottom whisker and bottom boundary of the box will be equal and the median line will be exactly centered with respect to the top and bottom box boundaries.

Examining the box plot for the Atlas V 401 vehicle shown in Figure 10, we can see that the fourth order fit provides slightly smaller range of absolute residuals than the third order fit but the spacing between the box segments are much more even, indicating a more symmetric data distribution. The degree of improvement between the third and fourth order boxes justifies the increase in equation complexity from a third to a fourth order polynomial in order to improve the quality of fit. Conversely, the fifth order box indicates that choosing the fifth order polynomial will not significantly decrease the residual range or improve the symmetry of the residual distribution and therefore the increase in equation complexity cannot be justified by the improvement in quality of fit. Hence the fourth order polynomial appears to provide the smallest range of residuals and the most normal distribution of data without adding unnecessary equation complexity. The above described process was repeated for launch vehicles sampled from several different families (Atlas V 401, Atlas V 431, Delta IV Medium, and H2A202-4S) to ensure that the fourth order fit would be appropriate for the vehicle data analyzed in this paper with the overall result that the fourth order polynomial would provide an adequate quality of fit without unnecessarily increasing equation complexity.

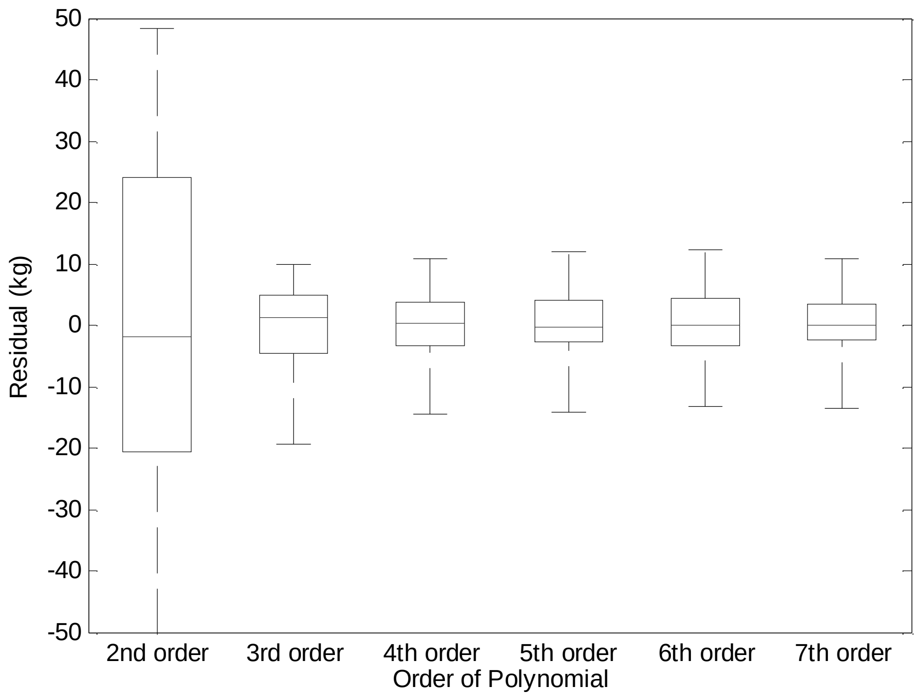

Fig. 10: Box Plot for the Atlas V 401 Vehicle.